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Engineering calculation procedure of critical compressive load of cone-shaped packages

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ABSTRACT

This paper presents experimental stability tests of packaging samples of cone- shaped plastic cups used in the food industry. The main theoretical principles of engineering methodology for evaluating the critical force of a cone-shaped plastic cup have been derived and validated. A mathematical model of critical load has been developed and studied with the aim of minimizing the package thickness.

1. Introduction

Development of modern packaging constructions requires consideration of numerous factors affecting their final characteristics and evaluation of their reliability, durability and. A special place among them is taken by thin-walled truncated cone-shaped packages (cups) whose characteristics are determined by the preset sizes and construction. The product material is also of great importance in determining the ability of cone-shaped cups to carry the load. All the previous experimental tests were based on analysing one particular characteristic of the packaging (cup). Typical characteristics of cone-shaped packages are: the slope angle, sidewall thickness, height and fillet radius. The empirical relations between the load-bearing capacity of a cone-shaped package and parameters mentioned above based on numerical simulations and experimental tests were analyzed by X. Ma et al. [1] and J. Hoffman [2]. However, a complete description of all the basic properties of a truncated plastic drinking cup under compression is possible only by modelling dependences of load carrying ability on all the main characteristics of the cup.

Experimental and analytical studies on the stability of conical shells have been developed by many other researchers. In addition, there were other pioneers such as Lackman and Penzien [3], Singer [4], Weingarten et al. [5], Lukasiewicz and Szyskowski [6] and Esslinger and Geier [7] who studied the stability of conical shell using different approaches. Several formulas for buckling load of a cone have been proposed for different contexts of the problem.

In the case of conical shells, there have been some experimental and analytical results, such as Singer [8], Thurston [9], Tani et al. [10], Petsios [11] and Pariatmono et al. [12], which suggested that the buckling load of the end-constrained cone is higher than the simply supported cone.

Compression test results of a common type plastic packaging construction are presented by Varžinskas et al. [13], which allow us to assess the impact of the package shape and construction on the packaging reliability and minimization of its mass.

Rodriguez in his paper [14] presents calculation methodology for the deformation of a sideways compressed polystyrene container by using the finite element method.

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Experimental study of plastic packages made of polystyrene (PS), used for packing granular and liquid products of different consistency, was presented by Kibirkštis et al. [15]. Graphic dependences of compression loads and deformations of the packages have been obtained and the maximum values of compression load have been determined, but the authors did not analyse the dependence of load carrying ability on all the main characteristics of the cup.

The analysis of the available research publications leads to the conclusion that the resistance of polystyrene packaging to compression has not been studied sufficiently.

The aim of the present work is modeling the dependence of critical load of cone-shaped packaging on its main characteristics. This study could lead to a new optimization procedure, allowing us to anticipate the initial packaging design so as to achieve the minimum thickness of the packaging under the preset critical load. The predicted minimum thickness would have a positive impact on minimization of package mass so reducing the amount of plastic material needed.

2. Modelling of critical loads

A number of papers [16-20] deal with the stability of cone-shaped shells. Axial compression of a conical shell was studied by Sachenkov in [20]. It is considered that the loss of stability is local and is accompanied by small waves appearing in the section $x = x_1$ or $x = x_2$ (Fig. 1 a). The formula for the critical load is:

$$P_{cr} = C_1 \frac{2\pi E\delta^2}{\sqrt{1-\nu^2}} \left[1 + \frac{C_1}{C_2} (1-\nu^2)^{1/4} \frac{\delta}{x_i} \left(\frac{x_i tg\alpha}{\delta} \right)^{3/2} \right]^{-1}, i = 1,2, \quad (1)$$

where ν and E are Poisson's ratio and modulus of elasticity of the material that is used for making the conical shell; δ is the shell thickness; 2α is the angle of conicality between the shell height and the generatrix, x_2 и x_1 are distances along the generatrix from the apex to the upper and lower base, respectively (Fig. 1 a), P_{cr} is full axial compression load. According to linear theory, coefficient $C_1 = 0.605$ (according to non-linear theory, $C_1 = 0.18$), $C_2 = 12/9$.

Following the results of the investigation by Seide [16] and Pilkey [17] on critical load of a truncated conical shell subjected to concentrated axial compression force,

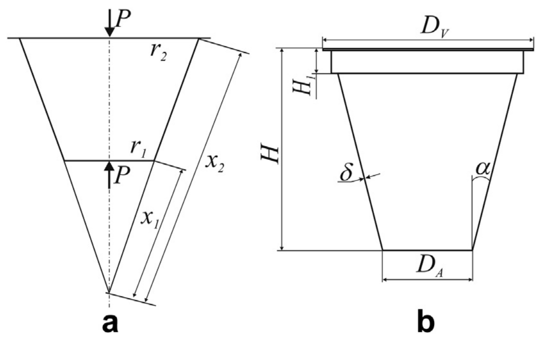

Fig. 1. Truncated conical package under axial compression (a) and geometrical dimensions of a truncated conical package (b) where H – package height; H_1 – rim height; D_V – package upper diameter; D_A – package bottom diameter; δ – package sidewall thickness; α – package sidewall slope angle.

the formula is presented for axisymmetrical buckling, simply supported at upper and lower edges of a frustum of cone

$$P_{cr} = \frac{2\pi E\delta^2}{\sqrt{3(1-\nu^2)}} \cos^2 \alpha \quad (2)$$

and for asymmetric buckling

$$P_{cr} = \frac{2\pi E\delta^2 \cos^2 \alpha}{\sqrt{3(1-\nu^2)}} \sqrt{\frac{1}{2} \frac{1 + r_1/r_2}{1 - r_1/r_2} \log \frac{r_2}{r_1}} \quad (3)$$

Following the above mentioned studies [16–20], the structural formula of critical loads for cone-shaped cups with possible technological changes (Fig. 1b) under axial compression in the cases of general and local loss of stability can be written as follows:

$$P_{cr} = K_c \chi(t, t_1, t_A, \gamma) = K_c \chi \left(\frac{H}{\delta}, \frac{H_1}{\delta}, \frac{D_A}{\delta}, \frac{\pi\alpha}{180} \right), K_c = \frac{2E\delta^2}{\sqrt{3(1-\nu^2)}}, \quad (4)$$

where $t = H/\delta$ is the parameter of height; $t_1 = H_1/\delta$ is the parameter of height of the upper technological rim; δ is the shell thickness; $t_A = D_A/\delta$ is the parameter of the cup bottom or index of wall thinness; $\gamma = \pi\alpha/180$ is the angle of conicality between the shell height and the generatrix measured in radians, α is the angle measured in degrees (half angle of cone); D_A, D_{V2} are the diameters of the lower and upper bases of the frustum of a cone, respectively, $D_{V2} = D_A + 2(H - H_1)tan\gamma$; $\chi(t, t_1, t_A, \gamma)$ is the correction of the function obtained from the experiment for determining general and local loss of stability.

The structural formula leads to the conclusion that during the experiment four parameters have changing values: t, t_1, t_A, γ , while the momentum and non-linearity of the pre-critical state is considered automatically. However, analysis of all the possible versions is labour-consuming and not always feasible. Therefore, it is necessary to develop appropriate methodology for presenting experimental findings in a compact way and minimizing their number.

The performed tests confirmed that there exist two forms of loss of stability: local, during which the first deformations appear in the shell, and general, when the shell loses its load-carrying strength completely.

The choice of variables was made on the basis of the theoretical-experimental method with regard to Formula (4). Four independent variables were chosen as determinants: $t = H/\delta$, $t_1 = H_1/\delta$, $t_A = D_A/\delta$, $\gamma = \pi\alpha/180$.

Following (4), we formulate the mathematical model:

$$\chi = \frac{P_{cr}}{K_c} = C_0 \left(\frac{H}{\delta} \right)^A \left(\frac{H_1}{\delta} \right)^B \left(\frac{D_A}{\delta} \right)^C \left(\frac{\pi\alpha}{180} \right)^D \quad (5)$$

The expression can be presented as follows:

$$\ln \left(\frac{P_{cr}}{K_c} \right) = \ln C_0 + A \ln \left(\frac{H}{\delta} \right) + B \ln \left(\frac{H_1}{\delta} \right) + C \ln \left(\frac{D_A}{\delta} \right) + D \ln \left(\frac{\pi\alpha}{180} \right) \quad (6)$$

or