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Efficient kinetic schemes for steady and unsteady flow simulations on unstructured meshes

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Abstract

This paper presents efficient second-order kinetic schemes on unstructured meshes for both compressible unsteady and incompressible steady flows. For compressible unsteady flows, a time-dependent gas distribution function with a discontinuous particle velocity space at a cell interface is constructed and used for the evaluations of both numerical fluxes and conservative flow variables. As a result, a compact scheme on the unstructured meshes is developed. For incompressible steady flows, a continuous second-order gas-kinetic BGK type scheme is presented, for which the time-dependent gas distribution function with a continuous particle velocity is used on unstructured meshes. The efficiency of the schemes lies in the fact that the slopes of the flow variables inside each cell can be constructed using values of the flow variables within that cell only without involving neighboring cells. Therefore, even with the stencil of a first-order scheme, a high resolution method is constructed. Numerical examples are presented which are compared with the benchmark solutions and the experimental measurements.

Keywords: Compressible unsteady flows; Incompressible steady flows; Efficient second-order kinetic schemes; Unstructured meshes

1. Introduction

Great progress has been achieved in the area of computational fluid dynamics (CFD) in the past decades. The developments of advanced numerical algorithms have made the CFD a valuable and indispensable tool in the analysis of highly complex flow problems. However, the construction of highly accurate and reliable numerical methods is still under demand as the geometry and flow physics become more sophisticated.

In recent years, the development of Boltzmann-type schemes has attracted much attention. The success of such schemes has appeared in a wide range of engineering applications, see for example, [13,14,8,15]. Among the Boltzmann-type schemes, the equilibrium-flux method (EFM) has been intensively studied [17]. EFM is a flux splitting method and is also referred to as a kinetic flux vector splitting (KFVS) scheme [17]. In fact, the EFM and KFVS schemes are identical. The KFVS scheme is composed of two steps. Firstly, the free transport equation or the col-

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lisionless Boltzmann equation is solved in the gas evolution stage for the flux evaluation. Then, the collision part is implicitly implemented through the preparation of a new equilibrium state inside each cell at the beginning of next time step. With the inclusion of Boltzmann collision model in the flux evaluation process, the gas-kinetic BGK scheme has been proposed in [2,6,20]. The BGK scheme differs from the KFVS method mainly in the inclusion of particle collisions in the gas evolution stage. Instead of solving the collisionless Boltzmann equation, the particle transport in the BGK scheme is controlled by a real particle collision time, which is related to the physical dissipative coefficients. In other words, instead of using the numerical time step as the particle collision time in the KFVS scheme, the real physical collision is included in the BGK scheme, where the accurate Navier–Stokes solutions have been obtained (cf. [21,22]). Since the gas evolution process is associated with a relaxation process, i.e. from a non-equilibrium state to an equilibrium one, the entropy condition is always satisfied by the BGK scheme. In the smooth region, where the physical structure can be well resolved by the numerical cell size, the BGK scheme gives an accurate compressible Navier–Stokes solution. In the discontinuity region, a delicate dissipative mechanism in the BGK scheme generates a stable and crisp shock transition.

The gas-kinetic BGK scheme is a finite volume method which originally targets on the simulation of compressible flows. In order to further extend its applicability, in this paper we construct a new efficient second-order kinetic schemes for compressible unsteady and incompressible steady flows on unstructured meshes. Here, a second-order method means that the reconstructed equilibrium and non-equilibrium states have piecewise linear distributions in space. At the same time, as analyzed in [16], in the smooth region the time accuracy of the scheme is equivalent to the Lax–Wendroff method for the Navier–Stokes equations. For the sake of clear presentation, in this paper we will restrict the presentation of our schemes only in two-dimensional space. However, the schemes of this paper can be straightforwardly extended to the three-dimensional case without any essential difficulty.

The basic idea in the construction of our efficient kinetic schemes lies in the fact that from a time accurate gas distribution function at a cell interface, we can not only calculate the numerical fluxes, but also evaluate accurate flow variables. Therefore, based on the cell averaged conservative flow variables and their cell interface values, we can construct or update the slopes within a cell solely. In other words, even with a stencil of a first-order scheme, a high resolution scheme can be still constructed. In this way, we avoid using the flow variables from neighboring cells in the construction of limited slopes. This is different from a traditional finite volume method, especially for a high-order scheme (see [11,14] for example). So, using a compact stencil we are able to construct an efficient gas-kinetic BGK scheme, which is computationally cheap and easy in coding. As is well-known, sometimes it is very difficult to choose a suitable stencil to construct all slopes on unstructured meshes. For incompressible steady flow simulation, the numerical dissipation introduced in the gas-kinetic BGK scheme through discontinuities of flow variables at a cell interface needs to be eliminated. In this case, a continuous gas distribution function corresponding to an isothermal flow is constructed for both flux evaluation and flow variables update at a cell interface. This is similar to the Lattice Boltzmann approach, where the solution in the isothermal low Mach number limit is obtained.

This paper is organized as follows. In Section 2, the gas-kinetic BGK model for compressible and incompressible isothermal flows is presented. Section 3 is devoted to the construction of the efficient second-order kinetic schemes on unstructured meshes. Section 4 present numerical examples which demonstrate the efficiency and accuracy of the schemes in the simulation of compressible and incompressible flows. The last section is the conclusion.

2. BGK model for compressible and incompressible isothermal flows

The BGK model in two space dimensions can be written as

$$f_t + uf_x + vf_y = \frac{g-f}{\tau}, \tag{2.1}$$

where f is the gas distribution function and g is the equilibrium state approached by f, (u, v) is the particle velocity. Both f and g are functions of x, y, t, u, v and the internal variable ξ. The particle collision time τ is related to the viscosity coefficient.

Generally, the equilibrium state is a Maxwellian distribution

$$g = \rho \left(\frac{\lambda}{\pi}\right)^{\frac{K+2}{2}} \mathrm{e}^{-\lambda((u-u)^2+(v-v)^2+\xi^2)}, \tag{2.2}$$