Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Physics 227 (2008) 3141–3158
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

Flux limiting embedded boundary technique for electromagnetic FDTD

M.T. Bettencourt *,[1]

Air Force Research Laboratory, Kirtland Air Force Base, Albuquerque, NM 87117, United States

Received 27 February 2007; received in revised form 8 November 2007; accepted 15 November 2007
Available online 15 December 2007

Abstract

A general approach for incorporating embedded boundaries into an electromagnetic finite difference time domain (FDTD) code is presented. This algorithm is shown to satisfy Gauss's law and enforces no magnetic monopoles while maintaining a globally second-order result (first-order at physical boundaries), with no added time-step restriction. Theoretically predicted superior results are shown with an 11% time-step reduction from the Courant stability limit. This is achieved through a physics-based flux limiting scheme near physical boundaries. Stability, local truncation error and energy conservation analysis are also provided.
Published by Elsevier Inc.

Keywords: Electromagnetic; EM; Finite difference time domain; FDTD; Conformal boundaries; Embedded boundaries; Flux limiting

1. Introduction

The finite difference time domain (FDTD) technique for solving Maxwell's equations has been a very powerful tool in the simulation of electromagnetic phenomena since its introduction by Yee [1]. Computational efficiency, ease of implementation, second-order accuracy (in the absence of physical boundaries) and energy conservation are among the method's many positive qualities. Furthermore, this scheme can be extended to include plasmas with particle-in-cell (PIC) techniques. However, a key weakness of the scheme is the low order boundary representation of physical geometry which can drop the global accuracy of the method to first-order for many physical geometries.

There have been many attempts to mitigate the errors caused by stair-cased boundaries in FDTD simulations through both the use of unstructured grids and solving Maxwell's equations on cut cells embedded in an otherwise structured grid. While fully unstructured grids are the mainstay in frequency domain codes, they are making inroads into time domain applications both for finite vols [2], finite element methods [3] and for spectral element codes [4]. These methods offer the flexibility of unstructured gridding, however they are more

* Tel.: +1 505 853 4320; fax: +1 505 846 0417.
E-mail address: icepic@kirtland.af.mil
[1] Research supported by a Grant from the Air Force Office of Sponsored Research (AFOSR) Contract number F2960100D00550031.

0021-9991/$ - see front matter Published by Elsevier Inc.
doi:10.1016/j.jcp.2007.11.043

complicated to implement, can suffer from long-time instabilities [5] and the incorporation of particles into these simulation techniques [6] is an unresolved area of research.

Several modifications to the standard FDTD algorithm have been proposed which maintain the inherit simplicity in the bulk of the domain and require special attention only adjacent to physical boundaries. The first of these is the scheme of Dey and Mittra [7], where the time-step is reduced and fractional cells of a sufficient size are handled through the standard algorithm. The work by Yu and Mittra [8] removed the time-step restriction required by [7], however, as shown in [9], the Yu–Mittra algorithm has a lower level of accuracy than the Dey–Mittra scheme. Benkler et al. [9] proposed a modified scheme similar to Yu–Mittra which simplified the computation and incorporated a reduced time-step to improve accuracy through improved accounting of the magnetic field area. The Benkler scheme shows improved accuracy over the Dey–Mittra and Yu–Mittra schemes listed above. Contour methods originally proposed by Jurgens et al. [10] modify the standard FDTD by using the integral forms of Maxwell's equations near physical boundaries. These methods, like the Dey–Mittra scheme, require a reduction in time-step to maintain stability.

In addition to the schemes listed above there exist a class of schemes known as area borrowing algorithms [11]. These algorithms enlarge cell volumes over a threshold by moving volume from a full cell to a partial cell in order to stabilize small cells. This class of methods trade the numerical complexity of updating partial cells below the stability limit with the geometric complexity of creating a set of sufficiently large cells around a physical boundary, which can be intractable in three-dimensions.

This article presents a new procedure for representing conformal boundaries within an FDTD code. This work is inspired by the research of Berger and Leveque [12] and Colella et al. [13] where the fluxes into a computational cell are redistributed based on geometric considerations. Unlike the schemes listed above, individual fluxes are limited on a face-by-face basis instead of a cell based flux redistribution.

A brief review of the traditional FDTD method proposed by Yee [1] follows. Section 3 presents a detailed one-dimensional analysis the stability, accuracy and energy conservation properties of the proposed method. Section4 extends the one-dimensional algorithm to multiple dimensions. Results showing second-order accuracy are presented for simple cavity problems and qualitative results are given for more complex geometries.

2. Standard finite difference time domain

We start with Maxwell's equations shown in differential form,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{1a}$$

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J} \tag{1b}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1c}$$

$$\nabla \cdot \mathbf{D} = \rho \tag{1d}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{1e}$$

$$\mathbf{D} = \epsilon \mathbf{E} \tag{1f}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields respectively, $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic flux densities respectively and μ, ϵ, ρ and $\mathbf{J}$ are the material permeability, permittivity, charge and current densities. The speed of light, $c = \frac{1}{\sqrt{\mu\epsilon}}$, is defined for convenience. Values written as $\mathbf{V}$ are assumed to be vectors of the form (V_x, V_y, V_z).

The standard finite difference time domain (FDTD) solves these equations in a leap-frog method on a staggered grid as shown in Fig. 1. This leads to the standard update equation:

$$B^{n+\frac{1}{2}}_{z;i+\frac{1}{2},j+\frac{1}{2},k} = B^{n-\frac{1}{2}}_{z;i+\frac{1}{2},j+\frac{1}{2},k} - \frac{\Delta t}{\Delta x}\left(E^{n}_{y;i+1,j+\frac{1}{2},k} - E^{n}_{y;i,j+\frac{1}{2},k}\right) + \frac{\Delta t}{\Delta y}\left(E^{n}_{x;i+\frac{1}{2},j+1,k} - E^{n}_{x;i+\frac{1}{2},j,k}\right) \tag{2}$$