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Inversion of Robin coefficient by a spectral stochastic finite element approach

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Abstract

This paper investigates a variational approach to the nonlinear stochastic inverse problem of probabilistically calibrating the Robin coefficient from boundary measurements for the steady-state heat conduction. The problem is formulated into an optimization problem, and mathematical properties relevant to its numerical computations are investigated. The spectral stochastic finite element method using polynomial chaos is utilized for the discretization of the optimization problem, and its convergence is analyzed. The nonlinear conjugate gradient method is derived for the optimization system. Numerical results for several two-dimensional problems are presented to illustrate the accuracy and efficiency of the stochastic finite element method.

Keywords: Robin inverse problem; Spectral stochastic finite element method; Conjugate gradient method; Uncertainty quantification; Stochastic inverse problems

1. Introduction

Inverse problems arise naturally in assorted engineering design, control and identification problems, and inverse theory has witnessed great success during the past few decades. In this paper, we are interested in probabilistically calibrating the heat transfer coefficient, hereafter denoted as the Robin coefficient, of a Robin boundary condition, which models the convection between the conducting body and the ambient environment, from the boundary measurements of the temperature and heat flux.

The values of the Robin coefficient are of immense practical interest in thermal problems, such as the design of gas-turbine blades and nuclear reactors and the analysis of quenching processes [1]. But its accurate values are difficult to obtain experimentally since they depend strongly on at least twelve variables or eight non-dimensional groups [2]. Instead engineers seek to infer it from measured data, which leads naturally to the nonlinear inverse problem of estimating the Robin coefficient from boundary or interior measurements

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[3,4]. Several non-destructive evaluation methods also give rise to this inverse problem. In corrosion detection, the Robin coefficient could represent the corrosion damage profile [5,6]; and in the study of MOSFET semiconductor devices, it contains information about the contact resistance and location of the metal-to-silicon contact window [7].

Several numerical methods [6,8–12,14] have been proposed for the Robin inverse problem, in particular in the context of corrosion detection, among which the least-squares method [9–12] has received intensive investigations and it has been implemented in the boundary integral equation method [9,10], the boundary element method [11] and the finite element method [12]. The Robin inverse problem studied in the literature so far is deterministic in the sense that the stochastic nature of the measurement errors is not rigorously considered and analyzed, and these deterministic inverse techniques yield only a single estimate of the large ensemble of solutions that are consistent with the given data without quantifying the uncertainties in the inverse solution.

Uncertainties are ubiquitous due to imperfect data acquisition, modeling errors and solution errors. The data is always contaminated by the inherent measurement errors, and only known with certain confidence, e.g. bounds on the error or the standard deviation of the noise. The forward model may be imprecise due to the presence of unmodeled physics, inherent variability of the physical system and limited ability of building realistic models. This occurs naturally for complex models with significant microstructures, e.g. the permeability of porous media [15], the heat conductivity of composite materials and polymeric materials, thermo-mechanical properties of polycrystalline materials, and domains with rough surfaces. Finally, the model may be so complex that the numerical approximation commits considerable amount of numerical noise [16], which in turn can affect the inverse solution significantly [17]. Uncertainties can be described in several ways, depending on the amount of information available, e.g. evidence theory, fuzzy set theory, worst-case scenario analysis and probabilistic setting (see e.g. [18] and extensive references therein). In this paper, we focus on probabilistic description of uncertainties, where the uncertain input data is modeled as random variables, or more generally, as stochastic processes with given correlation structures, and the resulting mathematical model is often a stochastic partial differential equation [19].

These uncertainties would certainly affect the inverse solution, and rigorous quantification of their effect on the inverse solution would allow enhanced decision-making strategies and deeper insight into the physical problem. However, the numerical analysis of inverse problems under uncertainties remains largely unexplored due to unprecedented computational challenges associated with stochastic numerics. Therefore, the development of computationally efficient and mathematically founded stochastic inverse methods that could account for uncertainties and that are able to provide the inverse solution with its uncertainties quantified is compelling.

Recent efforts on numerical methods for stochastic inverse problems include the Bayesian inference approach [17] and the spectral stochastic approach [20]. The Bayesian inference approach [17] could provide a complete probabilistic description of the inverse solution, and it also alleviates the difficult problem of selecting a suitable regularization parameter via hierarchical Bayesian models. However, it could be computationally expensive for nonlinear inverse problems. The spectral stochastic approach [20] elegantly integrates the powerful and efficient variational method with the state-of-art uncertainty quantification techniques, and it has been applied to reconstructing the heat flux. But no regularization has been considered in the mathematical formulation [20]. Considering the ill-posedness of the inverse problem, regularization should be incorporated in the variational formulation in order to rigorously justify the formulation, its stability and discretization as well as the convergence of the discrete system.

Motivated by the encouraging numerical results reported in [20], we shall propose a spectral stochastic finite element method using polynomial chaos for the stochastic Robin inverse problem, with some regularization incorporated in the variational formulation of the considered inverse problem. We shall also provide a systematic mathematical justification of the algorithm. This seems to be the first work of the kind for a stochastic inverse problem. The outline of the paper is as follows. Section 2 describes the mathematical formulation of the inverse problem and its reformulation as an optimization problem by defining a certain functional. Section 3 investigates mathematical properties of the functional relevant to its numerical computations. Section 4 describes the conjugate gradient method implemented in the spectral stochastic finite element method for the numerical solution of the optimization problem, and analyzes the convergence of the stochastic finite element approximation. Numerical results for several two-dimensional problems with the input data given in terms of polynomial chaos expansion (PCE) coefficients are presented and discussed in