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Generalized finite compact difference scheme for shock/complex flowfield interaction

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ABSTRACT

A class of generalized high order finite compact difference schemes is proposed for shock/vortex, shock/boundary layer interaction problems. The finite compact difference scheme takes the region between two shocks as a compact stencil. The high order WENO fluxes on shock stencils are used as the internal boundary fluxes for the compact scheme. A lemma based on the property of smoothness estimators on a 5-points stencil is given to detect the shock position. There is no free parameter introduced to switch the compact scheme and the WENO scheme. Some numerical experiments are given and they demonstrate that the present scheme has low dissipation due to the compact central differencing scheme used in the smooth regions.

1. Introduction

The numerical schemes for direct numerical simulation (DNS) and large eddy simulation (LES) of interaction of shock waves/complex flows need to resolve both shock waves and fine flow structures. Since weighted essentially non-oscillatory (WENO) schemes tend to have uniform higher order accuracy in smooth region and keep the essentially non-oscillatory properties near shock waves, they are widely used in simulation of flows with discontinuities such as shock waves and contact surfaces [1]. However, even though the order of accuracy for WENO schemes can be designed to be arbitrarily high [2,3], the resolution of short waves of WENO schemes in smooth regions tends to be more diffusive than the central differencing schemes with an equivalent order of accuracy.

The most accurate method to simulate wave dominated problems is the spectral method [4], but it is limited to simple geometries with generally periodic boundary conditions. Due to compact schemes' spectral-like resolution and their flexibility, compact schemes [5] have attracted a lot of attention and have been widely used for DNS and LES of turbulence flows [6–11]. A drawback of compact schemes is that they will generate Gibbs-like oscillations around shock waves or large gradients.

In recent years, there are many efforts to make compact schemes possess shock-capturing capability. Cockburn and Shu [12] develop the nonlinear stable compact schemes using the TVDM (total variation diminishing in the means) property. The schemes require an implicit symmetric matrix and a reconstruction from the mean variable obtained by TVDM compact schemes. Ravichandran [13] improves this type of schemes with a class of the compact upwind schemes developed without the limitation of a symmetric matrix. Tu and Yuan [14] construct a fifth-order shock-capturing compact upwind scheme by using a characteristic-based flux splitting limited method.

Adams and Shariff [15] propose a hybrid compact-ENO scheme for shock-turbulence interaction problems. Following the same basic approach, Pirozzoli [16] derives a conservative hybrid compact-WENO scheme. Ren et al. [17] present a

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fifth-order conservative hybrid compact-WENO scheme for shock-capturing calculation, which is constructed through the weighted average of conservative compact scheme and WENO scheme. Zhou et al. [18] suggest a new family of high order compact upwind difference schemes, which are later made to have shock-capturing capability by combining them with WENO schemes.

Shen et al. [19] propose a finite compact scheme, which treats the discontinuity as the internal boundary and avoids the global dependence of the traditional compact schemes. Combined with the TVD of ENO limiters, a set of high resolution finite compact (FC) difference schemes with only bi-diagonal matrix inversion are constructed [19,20].

Some of the above methods need to calculate the preliminary fluxes first by using a standard compact scheme [12–16]. This may result in contaminated (oscillatory) fluxes in the regions near discontinuities, and hence the compact fluxes will lose their high order accuracy in those regions. Some other methods using limiter functions will degrade the accuracy at extrema to first order [12–14,19]. All the hybrid schemes [12,15–19] introduce a free parameter to judge the flow gradient and to switch to the ENO/WENO schemes at discontinuities. Such parameters are usually problem dependent and hence lose their generality. The weighted compact nonlinear schemes proposed by Deng and Zhang [21] need to use more nodes than a standard compact scheme and hence lose the compactness. The same problem exists in the higher order extensions [2] of Deng and Zhang's method.

Artificial dissipation and compact filters are also introduced into compact schemes to help stabilize numerical solutions and reduce oscillations near discontinuities [11,22–28]. Nonlinear characteristics-based (artificial compression method, ACM) filters is used to construct the low-dissipative high order shock-capturing scheme by Yee et al. [29] with a problem dependent parameter introduced. A WENO-type smoothness estimator is used in [30] as a sensor to switch between the high-order compact spatial filters and the ACM filters, for which a threshold parameter for the sensor is also needed.

For the WENO schemes first proposed by Liu et al. [31] and improved by Jiang and Shu [32], a small difference between the smoothness estimators can reduce the numerical accuracy. A function to decrease the weights sensitivity to the smoothness estimators variation is suggested by Shen and Zha [33] to improve the WENO scheme accuracy. In addition, when it is applied to practical flows, the original value of the parameter ε used to construct the weight functions will generate oscillatory weights, which result in convergence and accuracy problems. An optimized ε value of 10^{-2} is suggested by Shen et al. [34] to improve convergence and accuracy. On the other hand, Henrick et al. [35] point out that the original smoothness indicators of Jiang and Shu fail to improve the accuracy order of a WENO scheme at critical points, where the first derivatives are zero. A mapping function is proposed by Henrick et al. [35] to obtain the optimal order near critical points. Recently, Borges et al. [36] suggest to use the whole 5-points stencil to devise a smoothness indicator of higher order than the classical smoothness indicator proposed by Jiang and Shu [32]. Shen and Zha [37] apply these smoothness estimators to improve accuracy at transition points of a discontinuity.

Based on the results of Borges et al. [36], this paper gives a lemma about the property of smoothness estimators on the whole 5-points stencil. A class of generalized finite compact difference schemes without any free parameters introduced is proposed by using the lemma to detect the shock region, in which a WENO scheme is used to calculate the fluxes.

2. The numerical algorithm

For the hyperbolic conservation law in the form

$$\frac{\partial u}{\partial t}+\frac{\partial f}{\partial x}=0 \quad (1)$$

the flux function $f(u)$ can be split into two parts as $f(u)=f^+(u)+f^-(u)$ with $df^+(u)/du \geqslant 0$ and $df^-(u)/du \leqslant 0$. The semi-discretization form of (1) can be written as

$$\frac{du_i(t)}{dt}=-\frac{1}{\Delta x}\left(h_{i+\frac{1}{2}}-h_{i-\frac{1}{2}}\right) \quad (2)$$

where the numerical flux is $h_{i+1/2}=h^+_{i+1/2}+h^-_{i+1/2}$. In this paper, only the positive part $h^+_{i+1/2}$ is described and the superscript "+" is dropped for simplicity. The $h^-_{i+1/2}$ is evaluated following the symmetric rule about $x_{i+1/2}$.

2.1. Weighted essentially non-oscillatory (WENO) scheme

The flux $h_{i+\frac{1}{2}}$ of the classical fifth-order WENO scheme [32,36] is built through the convex combination of interpolated values $\hat{f}^k\left(x_{i+\frac{1}{2}}\right)(k=0,1,2)$, in which $\hat{f}^k(x)$ is the third degree interpolation polynomial on stencil $S^3_k=(x_{i+k-2},x_{i+k-1},x_{i+k})$,

$$h_{i+\frac{1}{2}}=\sum_{k=0}^{2}\omega_k\hat{f}^k\left(x_{i+\frac{1}{2}}\right) \quad (3)$$

where

$$\hat{f}^k\left(x_{i+\frac{1}{2}}\right)=\hat{f}^k_{i+\frac{1}{2}}=\sum_{j=0}^{2}c_{kj}f_{i+k-2+j},\quad i=0,\ldots,N \quad (4)$$