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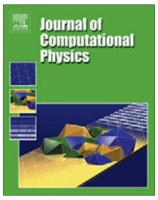

A class of finite difference schemes with low dispersion and controllable dissipation for DNS of compressible turbulence

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ABSTRACT

In this paper, a class of finite difference schemes which achieves low dispersion and controllable dissipation in smooth region and robust shock-capturing capabilities in the vicinity of discontinuities is presented. Firstly, a sufficient condition for semi-discrete finite difference schemes to have independent dispersion and dissipation is derived. This condition enables a novel approach to separately optimize the dissipation and dispersion properties of finite difference schemes and a class of schemes with minimized dispersion and controllable dissipation is thus obtained. Secondly, for the purpose of shock-capturing, one of these schemes is used as the linear part of the WENO scheme with symmetrical stencils to constructed an improved WENO scheme. At last, the improved WENO scheme is blended with its linear counterpart to form a new hybrid scheme for practical applications. The proposed scheme is accurate, flexible and robust. The accuracy and resolution of the proposed scheme are tested by the solutions of several benchmark test cases. The performance of this scheme is further demonstrated by its application in the direct numerical simulation of compressible turbulent channel flow between isothermal walls.

1. Introduction

Turbulent flows are characterized by a large range of length scales. To be reliable, a direct numerical simulation (DNS) of such flows must resolve these scales, especially the small ones with accuracy in both amplitude and phase. Therefore, dissipation and dispersion properties of numerical schemes are of crucial importance. Because of their superior spectral properties, spectral methods and compact schemes are extensively used [1–5]. However, these methods are limited to compute flows without shock waves since they have been found to cause non-physical oscillations when applied to flow with discontinuities. ENO and WENO schemes [6,7], on the other hand, provide robust shock-capturing capability and high order accuracy. However, in their original forms, the dispersion and dissipation properties of these schemes are not optimized. The 5th WENO scheme of Jiang and Shu, for instance, is reported too dissipative for the detailed simulation of turbulent flow [8]. A lot of efforts have been therefore devoted to developing numerical schemes with high resolution and good shock-capturing capabilities.

The observations presented above suggest that, for designing such schemes, a natural choice is to combine the ENO/WENO scheme with another scheme with spectral-like resolution to form a so-called hybrid scheme. Adams and Shariff [9] proposed the hybrid compact-ENO scheme that coupled a non-conservative compact scheme with a shock-capturing ENO scheme for shock and turbulence interaction simulation. The compact scheme is applied where the flow field is smooth

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and the ENO scheme is used near the discontinuities. Pirozzoli [10] derived a hybrid compact-WENO scheme in which a conservative compact scheme is coupled with a WENO scheme to make the overall scheme conservative. Ren et al. [11] improved the previous hybrid compact-WENO scheme by designing a continuous weight function to avoid the abrupt transition between the compact and WENO schemes. When solving the system of hyperbolic conservation laws, they used the characteristic decomposition to improve the resolution of the scheme. Shen and Yang [12] have also developed a hybrid compact-WENO scheme. Similar ideas have been adopted by Kim and Kwon [13], who proposed a high-order hybrid scheme which combined a central scheme and the WENO scheme for compressible flow field analysis. Costa and Don [14] developed the hybrid spectral-WENO scheme.

An alternative approach is to improve the spectral properties of the shock-capturing schemes by using the optimization techniques. The optimization procedures are usually based on the pioneering works of Tam and Webb [15], who devised the dispersion-relation-preserving (DRP) scheme for computational acoustics. Lockard et al. [16] developed an optimized ENO scheme for the solution of Euler equations. Their simulations showed that the optimized ENO scheme performed better than the non-optimized one for the case of linear wave propagation problems. Weirs and Candler [17] developed optimized WENO schemes to solve the hyperbolic conservation laws. They devised a strategy that consists of optimizing the weights of all candidate reconstructions and adding an additional candidate stencil to make the stencils symmetric rather than upwind-biased. This idea was further explored by Martín et al. [8], who developed a bandwidth-optimized WENO scheme for the DNS of compressible turbulent boundary layer. The essential idea of bandwidth-optimized WENO scheme is to determine the optimal weight by minimizing an elaborately designed integrated error function. Wang and Chen [18] proposed the optimized WENO scheme for solution of the linearized Euler equations with discontinuities. They designed a two level optimization procedure but only considered the biased stencils. Ponziani et al. [19] used the optimized WENO scheme for the DNS of isotropic compressible turbulence as well as aeroacoustic phenomena.

Concerning the optimization of the spectral properties of the finite differences schemes, it is generally accepted that the dispersion error should be minimized according to some chosen criteria. However, there are no general guidelines on how the dissipation should be optimized. A scheme with very large numerical dissipation is surely not suitable for DNS. On the other hand, Lechner et al. [20] noticed that the minimal dissipation produced by central difference scheme was insufficient in suppressing the numerical oscillation and could lead to instability. Therefore, a small amount of dissipation is needed to suppress numerical instabilities that may be caused by unresolved high wavenumber structures of the solution [20]. This point was further confirmed by Pirozzoli [10] who pointed out that a certain amount of dissipation was not necessarily a bad feature since, in the range of high wavenumbers, the waves propagated at an incorrect speed. It was then desirable to damp them as much as possible. The main difficulty in the optimization of the dissipation properties of the finite difference schemes is that the optimal dissipation is often problem dependent. For example, the bandwidth-optimized WENO scheme derived by Martín et al. [8] gave satisfying results in the DNS of supersonic boundary layer. However, this scheme is susceptible to cause numerical oscillations in some test cases as reported by Cai and Ladeinde [21].

According to the above discussion, it would be beneficial to design high order numerical schemes with minimized dispersion and controllable dissipation. However, this is not always possible since the dissipation and dispersion properties of a finite difference scheme are often inter-dependent. Indeed, in most of the available procedures for the optimization of the spectral properties of the finite difference schemes, the cost functions are the blending of the dissipation and dispersion errors. As a result, the change of the dissipation properties may deteriorate the already optimized dispersion properties. Therefore, it is desirable to have a class of finite difference schemes in which the dispersion and dissipation can be controlled separately.

In the present paper, we will design a class of semi-discrete finite difference schemes with minimized dispersion and controllable dissipation which is called the MDCD schemes hereafter. To design a MDCD scheme, a sufficient condition for the semi-discrete schemes to have independent dispersion and dissipation is derived, which makes it possible to optimize the dispersion properties of the scheme and to adjust the dissipation by the introduction of a free parameter. An important feature of the MDCD schemes is that the adjustment of the dissipation will not affect the optimized dispersion properties of the schemes. It is also found that the linear MDCD schemes can be used as the linear part of the WENO schemes to construct the so-called MDCD–WENO schemes which can be used to compute flow with discontinuities. As noted by Pirozzoli [22] and Shen et al. [23], the nonlinear mechanisms of the shock-capturing schemes may cause a dramatic corruption of their spectral properties even for the smooth flows. In the mean time, it is noted that the nonlinear weighting procedure of the WENO scheme is need only in the vicinity of discontinuities. Therefore, a hybrid scheme is proposed which is the blending of the linear MDCD scheme and the MDCD–WENO scheme using the technique of Ren et al. [11]. The approximate dispersion relation (ADR) [22] is computed to study the spectral properties of the nonlinear hybrid scheme and significant improvements over the original and optimized WENO schemes are observed. It is worthwhile to point out that the MDCD properties not only can improve the resolution of the finite difference schemes, but also enhance their flexibility and robustness. Because the dissipation of the MDCD schemes is adjustable, it is possible to make the MDCD schemes to posses smallest possible dissipation that is sufficient to ensure the stability and the damping of spurious numerical oscillations by adjusting only one parameter. On the other hand, for some very demanding problems with strong discontinuities, a rather large dissipation can be provided to enhance the robustness of the MDCD scheme so that a stable and converged numerical result can be obtained. Several benchmark test cases are presented in the present paper to demonstrate the superior performance of the proposed hybrid scheme. This scheme has also been applied successfully to the DNS of compressible turbulent channel flows.