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Property modelling

Poisson's ratio and the incompressibility relation for various strain measures with the example of a silica-filled SBR rubber in uniaxial tension tests

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ABSTRACT

The controversy in the definition of Poisson's ratio (PR) as a material constant is discussed in this study. PR of an isotropic material is usually defined as the ratio, taken with the opposite sign, between its lateral and longitudinal strains under the action of longitudinal stresses. However, if deformations of the material are large, the value of PR depends on the strain measure used. Five different measures of strain are considered, and a unified relation in terms of stretch ratios is obtained for calculating the PR. It is demonstrated that only for Hencky strains is the value of PR of an incompressible material constant and equal to 0.5 over its entire extension range. Other measures lead to stretch-dependent PRs. A generalized relation for the volume strain is found in terms of strain invariants. With the example of uniaxial tension of a silica-filled rubber, the Cauchy and Hencky strains are used to demonstrate different ways for checking the incompressibility of a material and the evaluation of PR. The level of incompressibility of the rubber and its practical importance are evaluated.

1. Introduction

The deformation behaviour of a homogeneous, isotropic, linearly elastic material can be characterized by only two physical constants: the elastic modulus and Poisson's ratio (PR), which can be determined in a simple tension test [1]. The elastic modulus and its changes with various factors is widely considered in the literature for different kinds of materials, but there is still a limited number of papers on their PR. With the advent of modern noncontact methods of strain measurement, the accuracy of measured lateral contraction in tension tests and, hence, the determination of PR has been significantly improved, and the measuring procedure made much simpler [2–7]. However, in spite of the relatively simple acquisition of initial data, the interpretations of experimental results can be contradictory.

It is commonly assumed that the PR is a material constant and, hence, is independent of strain. Some authors contend that this is true only at small strains. Others insist on the use of the "mathematically correct" and "true" definition of PR based on the Hencky measure of strains when dealing with large deformations. In reality, depending on the strain measure used for characterizing the deformation, the value of PR and its variation with growing strain can be different. It is a question of individual preference which measure to use for solving a specific problem.

The deviation of PR from its initial value is often related to changes in the microstructure of a material, which can be evaluated from variations in its volume during deformation [8–14]. For example, the increase in volume can be associated with the formation of microdefects and growing void content in a material. The relaxation and orientation effects, in turn, manifest themselves in decreasing volume and increasing PR with growing extension [6,10,11,15]. It will be shown in this study that the strain dependence of PR may not be necessarily caused by volume changes, i.e.,

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by structural changes in a material, but can also be related to the specific strain measure used.

In its common formulation, the PR is defined as the ratio, taken with the opposite sign, between the lateral and axial strains in uniaxial tension or compression. It was originally defined for "rigid" materials – metals, which undergo small deformations and do not show time-dependent effects [1]. For "soft" materials, such as many polymers, rubbers, soft biological tissues etc., exhibiting large deformations accompanied in addition by relaxation, the traditional definition of PR can lead to contradictory results. To characterize the deformation behaviour of such materials over a wide range of strains and to emphasize that the PR in its original formulation is not a constant, some authors use the term "lateral contraction ratio" or introduce the concept of "Poisson function" [6,15–21]. The latter is especially widely used when dealing with materials with a negative PR – auxetics [19,20]. The time dependence of PR, which is commonly studied by introducing the concept of a "viscoelastic PR", are widely considered in the literature in both theoretical and experimental aspects [6,15–18]. In the present study, we will deal only with elastic deformations, and time dependences will be not considered. The emphasis will be placed on the behaviour of highly deformable nearly incompressible materials – rubbers.

Rubbers are employed in a wide variety of different industrial and automotive areas. The mechanical characteristics of rubbers can be highly improved by incorporating small-size filler particles into the cross-linked elastomer matrix, such as carbon-black or silica particles [22–24]. Filled rubbers can exhibit very large deformations, which exceed those of the neat rubber tenfold. This effect, which comes from the filler–filler and filler–rubber interactions, both chemical and physical, and take place on different length scales, is classically divided into different types: (i) the hydrodynamical effect associated with the inclusions dispersed in the matrix, (ii) the strong interactions between the rubber and filler, and (iii) the filler networking contribution. At small strains, most rubbers are incompressible or nearly incompressible. Hence, their volume strain tends to 0, but the PR is close to 0.5. However, at large stretch ratios, the incompressibility assumption can be violated due to the formation of defects in the material microstructure [14]. The level of its defectiveness depends of the nature and amount of fillers, since rigid particles facilitate the formation of voids at the filler-matrix interface [10,13]. Despite the well-known debonding phenomenon, only a limited number of papers are devoted to determining the limit of incompressibility of filled rubbers and variations in the PR. It is worthy of note that the evaluation of volume is important in calculating the true stresses, when an unjustified incompressibility assumption can lead to erroneous results.

The aim of the present study is to show alternative ways for defining the PR of isotropic homogeneous elastic materials in the regions of small and large deformations and to correlate them with volume changes. To give an insight into the problem, some fundamental definitions are expounded in the study. The reasons for inconsistencies in the interpretation of theoretical and experimental data are discussed with the example of a silica-filled styrene butadiene (SBR) rubber subjected to uniaxial tension. The level of incompressibility of the material and its practical importance are evaluated.

2. Definitions

2.1. Measure of strain

Strain is a dimensionless quantity, introduced to describe the deformation of bodies [25,26]. It is not a measurable physical quantity and can be defined in various ways. Some of possible strain definitions will be given here to appreciate their differences.

Let us consider the uniaxial tension of a homogeneous isotropic bar (Fig. 1), with its unstressed state taken as the reference state.

The basic dimensionless measure of its axial deformation, having a clear physical interpretation, is the stretch ratio λ_1, also called the extension ratio, defined as the length of the bar in the current state, l, divided by its length in the reference state, l_0:

$$\lambda_1 = \frac{l}{l_0} \tag{1}$$

We will relate different strain definitions to this basic quantity.

The most common definition of axial strain of the bar is the Cauchy strain ε_1^C (called also the nominal or engineering strain), which is expressed as the ratio between the total axial deformation $l - l_0$ and its length in the reference state and, with account of Eq. (1), can be presented in the form

$$\varepsilon_1^C = \frac{l - l_0}{l_0} = \lambda_1 - 1 \tag{2}$$

Potentially, any function of λ_1 can be used as a strain measure if it is equal to zero at $\lambda_1 = 1$ and reduces to the strain defined by Eq. (2) when $\lambda_1 - 1$ is infinitesimal. These conditions are satisfied by the strain measures defined by Green, Almansi, Swainger, and Hencky [26–28], namely

$$\varepsilon_1^G = \frac{l^2 - l_0^2}{2l_0^2} = \frac{1}{2}\left(\lambda_1^2 - 1\right)$$

$$\varepsilon_1^A = \frac{l^2 - l_0^2}{2l^2} = \frac{1}{2}\left(1 - \lambda_1^{-2}\right)$$

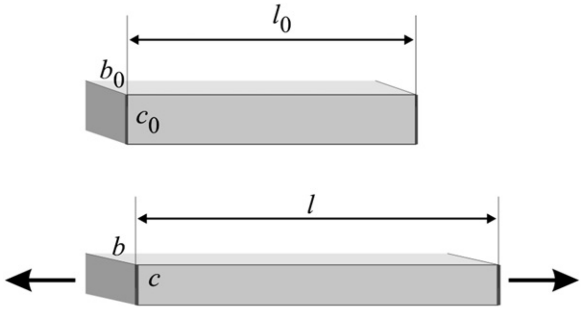

Fig. 1. A bar in uniaxial tension.