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Numerical simulation of non-viscous liquid pinch-off using a coupled level set-boundary integral method ☆

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ABSTRACT

Simulations of the pinch-off of an inviscid fluid column are carried out based upon a potential flow model with capillary forces. The interface location and the time evolution of the free surface boundary condition are both approximated by means of level set techniques on a fixed domain. The interface velocity is obtained via a Galerkin boundary integral solution of the 3D axisymmetric Laplace equation. A short-time analytical solution of the Raleigh–Taylor instability in a liquid column is available, and this result is compared with our numerical experiments to validate the algorithm. The method is capable of handling pinch-off and after pinch-off events, and simulations showing the time evolution of the fluid tube are presented.

1. Introduction and overview

A significant challenge in the numerical solution of free boundary problems is when the domain undergoes topological changes. This is often the case for the potential flow models that describe a variety of important fluid flow problems, the Rayleigh–Taylor (or Rayleigh–Plateau) instability of a fluid column considered herein being a prime example [26,27]. A second critical aspect of these (and other) simulations is that, on the free surface, the boundary condition for the Laplace equation must be obtained by solving a separate partial differential equation defined on the evolving front.

The Level Set Method was specifically designed to cope with topological changes in moving boundary problems [35]. Moreover, for advancing material properties defined and governed by a differential equation on the front, effective Level Set techniques have been recently developed [3]. As the Level Set approach produces (almost directly) a new surface mesh if desired, it invites solving the governing equation in the volume by means of a Boundary Integral analysis. These combined methods were first applied to successfully simulate complex dendritic growth in solidification [30]. More recent work has investigated the field emission of liquid droplets [38] and interface motion in two phase flows [8]. The Level Set algorithm for advancing the free boundary condition was initially employed to model the propagation and breaking of waves over

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sloping beaches [12,13]. In this work however, due to the limitation of the potential flow assumption, modeling of the wave after breaking (reconnection) was not attempted.

Drop formation problems, for viscous and non-viscous fluids, have been widely studied for many years due to its fascinating nature and interest in various technical and industrial fields, such as inkjet printing, sprays and electrosprays, etc. The first outstanding contributions were due to Savart [28], Plateau [26] and Rayleigh [27] and an extensive review of fluid break up has been given by Eggers [10,11]. Other interesting works in this field can be found in [4,6,18,21,24,29].

In this paper an inviscid fluid under the effect of capillary forces will be studied. Assuming that the liquid (*e.g.*, water) remains in the inviscid regime down to molecular scales, pinch-off and drop formation will result in a Rayleigh–Taylor instability. It has been shown theoretically [5,9] and computationally [19] that the phenomenon of inviscid pinch-off is asymptotically self-similar with both radial and axial length scales decreasing as $\tau^{2/3}$ and velocities increasing like $\tau^{-1/3}$, where τ is the time to pinch-off. It will be demonstrated that these results can be observed in the numerical simulations, validating the numerical methods.

A mathematical model and numerical approximation for the evolution of a 3D axisymmetric fluid domain is presented, capturing the time evolution of a fluid column before and after pinch-off events. The algorithm is capable of continuing the evolution of the first drops through the subsequent cascade of drop formation.

The paper is organized as follows: In Section 2 we present the model equations for an inviscid fluid flow in 3D and its axisymmetric version using a Lagrangian–Eulerian formulation. The complete Eulerian approach of the model equations using the Level Set Method is established in Section 3; we also demonstrate that the recasted system of PDE's automatically incorporates topological changes of the free surface and the evolution of the associated velocity potential function. In Section 4 we present the numerical schemes used, with a detailed description of the complete algorithm. Finally, in Section 5, we first present numerical results for the linearized model and compare them with the short-time analytical solution. Then, the full nonlinear approximation is used to compute the evolution of the fluid column before and after first pinch-off, following the satellite drop evolution and its subsequent break up. A series of numerical experiments are carried out to show the convergence of the algorithm. We complete the validation of the numerical results by checking the self-similar scaling laws for the first pinch-off, as well as the subsequent pinch-off occurrences.

2. The governing equations

To model the Rayleigh–Taylor problem, consider an infinite liquid column in the absence of gravity and initially at rest. Movement of the fluid is induced by perturbing the free surface of the cylinder with a small amplitude wave of wave number $k = \frac{2\pi}{L}$. For the numerical simulations, the domain will be made finite by introducing lateral boundaries for the cylinder and imposing periodic boundary conditions for these surfaces.

Let $\Omega(t)$ be the 3D cylindrical fluid domain surrounded by air and $\Gamma_t(\mathbf{s}) = (x(\mathbf{s},t), y(\mathbf{s},t), z(\mathbf{s},t))$ a parametrization of the free surface boundary at time t (see Fig. 1). For an incompressible and inviscid fluid, the governing equations are the Euler equations

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega(t), \tag{1}$$

$$\mathbf{u}_t + \mathbf{u} \cdot (\nabla \cdot \mathbf{u}) = \frac{-\nabla p}{\rho} + \mathbf{b} \quad \text{in } \Omega(t), \tag{2}$$

where $\mathbf{u}(x,y,z,t)$ is the fluid velocity, $p(x,y,z,t)$ the pressure field, $\mathbf{b}(x,y,z,t)$ the body forces (per unit mass), and ρ is the fluid density.

Further, if irrotationality is assumed, the vorticity vanishes everywhere in the flow. In this case, the Helmholtz decomposition states that the velocity field can be represented as the gradient of a scalar function, referred to as the velocity potential $\phi(x,y,z,t)$. Thus, $\mathbf{u} = \nabla\phi$, and the Euler equations can be written as

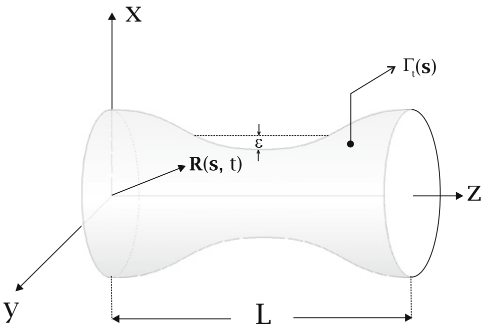

Fig. 1. Cylinder geometry in 3D.