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Test Method

On the effect of shear and local deformation in three-point bending tests

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Abstract

The three-point bending test is analysed taking into account shear and local deformation effects in the load application and supports. A new procedure is proposed in order to obtain flexural modulus, shear modulus and local deformation stiffness. Indentation tests and three-point bending tests at five different spans have been carried out on two specimens of T6T/F593 carbon/epoxy unidirectional composite. Since local deformation effects are not linear, slopes of load–deflection curves have been determined in three fixed strain ranges and a fixed load range. The best results have been obtained for the highest strain range, as local deformation effects can be considered linear. It has been seen that local deformation effects can have even more importance than shear effects in the case of small spans and large thickness.

Keywords: Three-point bending; Flexural modulus; Shear modulus; Local deformation; Indentation; Unidirectional composite

1. Introduction

Shear effects have influence in bending tests when the span-to-depth ratio is not large. In particular, in unidirectional composites, the effect is important when the test is carried out in the direction of the fibre, since the ratio between longitudinal flexural modulus and shear modulus is greater than 20. In the case of isotropic materials, the maximum ratio between longitudinal modulus and shear modulus is 3, due to the maximum value of Poisson ratio being 0.5.

Based on the fact that in the displacement of a three-point bending test both flexural modulus and shear modulus are involved, it is possible to calculate both of them by carrying out tests at different spans for the same specimen. Nevertheless, in the case where displacements are obtained from machine displacement, local deformation effects appear. According to Brancheriau et al. [1], the supports and loading head indentation effects are not negligible but have the same influence as the shear effect. The indentation depends on the material stiffness and the load level applied to the specimen. Jalali and Taheri [2] proposed the varying span method based on three-point bending tests at different span-to-depth, eliminating the effect of local deformation from the test results.

The standard ISO 14125 [3] proposes a fixed strain range to be used in modulus determination when shear effects are negligible. According to Mujika [4], the variation of span in three-point and four-point

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bending caused by bending rotations is related to the strain range used for determining flexural modulus. Correction factors have been proposed for taking into account the mentioned effect.

In the present work, a new approach for calculating middle point displacement taking into account flexural, shear and local deformation effects is proposed. The corrections related to flexural rotations are also included.

2. Displacement of the load application nose

Fig. 1 shows the model used for determining the displacement of the load application nose. Linear springs are considered as a first approach to model local deformations at reaction points and at the application nose, in spite of local deformation effects related to indentation not being linear [1,5]. The spring constants are termed k_1 for supports and k_2 for the load application nose.

The strain energy due to bending moments, shear forces and springs is given by

$$U = \int_L \frac{M^2}{2\,E_f I}\,\mathrm{d}l + \frac{6}{5}\int_L \frac{Q^2}{2\,GA}\,\mathrm{d}l + 2\frac{F_1^2}{2\,k_1} + \frac{F_2^2}{2\,k_2}, \tag{1}$$

where M is the bending moment; Q the shear force; F_1 and F_2 the forces at supports and load application nose, respectively; E_f the flexural modulus; G the shear modulus; I the moment of inertia with respect to the middle plane: $I = wh^3/12$; w the width of the specimen; and h is the thickness of the specimen.

According to Castigliano's second theorem, the displacement of the middle point of the specimen δ is

$$\delta = \frac{\partial U}{\partial P} = \int_L \frac{MM'}{E_f I}\,\mathrm{d}l + \frac{6}{5}\int_L \frac{QQ'}{GA}\,\mathrm{d}l + 2\frac{F_1 F_1'}{k_1} + \frac{F_2 F_2'}{k_2}, \tag{2}$$

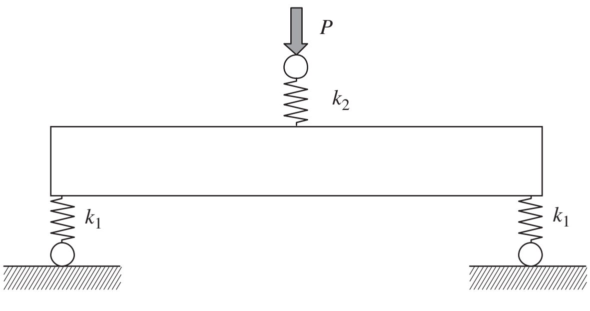

Fig. 1. Model of three-point bending test including local deformation effects.

where primes indicate derivatives with respect to P. Bending moments and shear forces in each half of the specimen, the spring forces and their derivatives are:

$$\begin{aligned} &0<x<\tfrac{L}{2}\left\{\begin{array}{ll} M=\frac{P}{2}x, & M'=\frac{1}{2}x,\\ Q=\frac{P}{2}, & M'=\frac{1}{2}, \end{array}\right.\\ &F_1=\tfrac{P}{2}, \qquad F_1'=\tfrac{1}{2},\\ &F_2=P, \qquad F_2'=1. \end{aligned} \tag{3}$$

Replacing Eq. (3) in Eq. (2) results in

$$\delta = \frac{PL^3}{4\,E_f wh^3} + \frac{3PL}{10\,Gwh} + \frac{P}{k}, \tag{4}$$

where $\frac{1}{k} = \frac{1}{2k_1} + \frac{1}{k_2}$.

The support span L is modified due to bending rotations at the ends [4]. L_0 being the initial span, Eq. (4) can be written as

$$\delta_C = \frac{PL_0^3}{4E_f wh^3}\left(1 - 6\frac{\theta R}{L_0}\right) + \frac{3PL_0}{10Gwh}\left(1 - 2\frac{\theta R}{L_0}\right) + \frac{P}{k}, \tag{5}$$

where θ is the angle at supports and R is the radius of the support cylinder. Relating the angle with the maximum strain ε, results in

$$\left.\begin{array}{l} \theta = \frac{3PL_0^2}{4E_f wh^3}\\ \varepsilon = \frac{3PL_0}{2E_f wh^2} \end{array}\right\} \frac{\theta}{\varepsilon} = \frac{L_0}{2h}. \tag{6}$$

In Eq. (6), the difference between tensile and compressive moduli [6] has not been considered for strain calculation. It is assumed that $E_t = E_C = E_f$, where E_t and E_C are tensile and compressive moduli, respectively.

Replacing the relation given in Eq. (6) in Eq. (5) and after rearranging terms in order to analyse the relative effect of shear and local deformations, Eq. (5) can be written as

$$\delta_C = \frac{PL_0^3}{4E_f wh^3}\left[\left(1 - \frac{3R}{h}\varepsilon\right) + \frac{6}{5}\frac{E_f}{G}\left(\frac{h}{L_0}\right)^2 \times \left(1 - \frac{R}{h}\varepsilon\right) + 4\frac{E_f w}{k}\left(\frac{h}{L_0}\right)^3\right]. \tag{7}$$

3. Determination of E_f, G and k

After calculating the difference between displacements at two points 1 and 2, flexural modulus