Journal of Computational Physics 231 (2012) 328–344

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

From the paddle to the beach – A Boussinesq shallow water numerical wave tank based on Madsen and Sørensen's equations

Jana Orszaghova *, Alistair G.L. Borthwick, Paul H. Taylor

Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

ARTICLE INFO

Article history:
Received 3 March 2011
Received in revised form 27 July 2011
Accepted 22 August 2011
Available online 5 October 2011

Keywords:
Boussinesq
Wave propagation
Runup
Overtopping
Piston paddle

ABSTRACT

This article describes a one-dimensional numerical model of a shallow-water flume with an in-built piston paddle moving boundary wavemaker. The model is based on a set of enhanced Boussinesq equations and the nonlinear shallow water equations. Wave breaking is described approximately, by locally switching to the nonlinear shallow water equations when a critical wave steepness is reached. The moving shoreline is calculated as part of the solution. The piston paddle wavemaker operates on a movable grid, which is Lagrangian on the paddle face and Eulerian away from the paddle. The governing equations are, however, evolved on a fixed mapped grid, and the newly calculated solution is transformed back onto the moving grid via a domain mapping technique. Validation test results are compared against analytical solutions, confirming correct discretisation of the governing equations, wave generation via the numerical paddle, and movement of the wet/dry front. Simulations are presented that reproduce laboratory experiments of wave runup on a plane beach and wave overtopping of a laboratory seawall, involving solitary waves and compact wave groups. In practice, the numerical model is suitable for simulating the propagation of weakly dispersive waves and can additionally model any associated inundation, overtopping or inland flooding within the same simulation.

1. Introduction

Numerical models based on Boussinesq-type equations are increasingly popular operational predictive tools in coastal engineering. These essentially two-dimensional models avoid free surface boundary issues by explicitly eliminating the vertical coordinate, yet retaining some vertical flow structure. Nowadays Boussinesq-type models are being applied to relatively large scale coastal problems (see for example [9,21]). It should be noted that there are other approaches used in practice based on non-hydrostatic shallow flow solvers (see [31,37]).

The classical Boussinesq equations derived by Peregrine [27] were limited to weakly nonlinear and weakly dispersive waves and assumed $O(\varepsilon) = O(\mu^2)$, whereby nonlinearity ε represents the ratio of wave amplitude to depth, and dispersion μ is the ratio of depth to wavelength. Since 1967, there have been considerable attempts at extending the applicability of Boussinesq equations both onshore and offshore. Comprehensive reviews of the earlier development of the field, including derivations, can be found in Dingemans [7] and Madsen and Schäffer [23]. Of the myriads of candidates, the enhanced equation set by Madsen and Sørensen [24] is $O(\varepsilon, \mu^2)$ accurate, yet has improved dispersion properties thanks to a mathematical manipulation of the dispersive terms. The incorporation of a Padé approximant of the linear dispersion relation into the momentum equations, as suggested by Madsen and Sørensen [24], results in equations suitable for water depth as deep

* Corresponding author.
E-mail addresses: jana.orszaghova@eng.ox.ac.uk, jana.orszaghova@gmail.com (J. Orszaghova).

0021-9991/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcp.2011.08.028

as $\mu = 0.5$ (approximately equivalent to $kh = 3$, with k representing the wavenumber and h is the still water depth). The vertical profile in this extended deeper region is not properly modelled, due to the underlying assumption of the quadratic vertical variation of the horizontal velocity, yet the wave propagation in the extended region is captured with reasonable accuracy. Another widely used equation set was derived by Nwogu [25], and possesses equivalent dispersion properties. Nwogu's equations are formulated in terms of the surface elevation and horizontal velocity at a specific depth chosen to minimise wave propagation errors from linear theory. Further extensions to the validity of Boussinesq-type equations have been achieved by deriving fully nonlinear equations, whereby $O(\varepsilon) = 1$ is assumed, as well as deriving higher order equations which retain $O(\mu^4)$ and equivalent terms (see for example [35,10,1,22]). The emphasis in much of this work is to extend the range of applicability of Boussinesq models into deeper water. In contrast, the present paper seeks to extend the applicability of a simple Boussinesq model to zero depth.

Boussinesq-type equations cannot model wave breaking, unless they are modified to account for the associated energy dissipation. The most common breaking treatments in the Boussinesq framework are the surface roller concept (see [29]) or the inclusion of artificial viscosity in the momentum equation (see [36,19]). These have been successful and facilitated the application of solvers to regions further inshore into the surf zone, but at the cost of requiring tunable parameters. On the other hand, the nonlinear shallow water equations are appropriate for modelling broken waves in the surf zone. Dissipation of energy is captured physically correctly (see textbooks on shallow water shock conditions, for example Section 2.7 in Johnson [18]). Due to the lack of frequency dispersion in the nonlinear shallow water equations, any waveform modelled by these equations has a tendency to shock up to produce bores or hydraulic jumps, even on a horizontal bottom. For this reason the equations cannot be used pre-breaking, as they would lead to incorrect and premature breaking.

The present paper describes a hybrid numerical model based on the Boussinesq equations derived by Madsen and Sørensen [24] pre-breaking and the nonlinear shallow water equations post-breaking. In this way the appropriate governing equations are applied at all stages as the wave propagates from intermediate to shallow water. Our hybrid model builds upon work of Borthwick et al. [4]. Recently there have been similar hybrid models proposed by Tonelli and Petti [32] and Bonneton et al. [3]. Although the chosen set of Boussinesq equations contains less of the physics and is relatively unsophisticated compared to the newer derivations (see above), it has been selected for its simplicity (small number of low order terms) and relative ease for numerical computation. In our model, the breaking criterion, which triggers a local switch from the Boussinesq to the nonlinear shallow water equations, is based on local wave steepness. An appropriate shock capturing numerical scheme is applied to the nonlinear shallow water equations in order to capture the propagating bores. A well-established Godunov-type finite volume method, with data reconstruction, is used, following Hu et al. [13], Hubbard and Dodd [14], and Liang and Borthwick [20] among others. Such methods additionally allow for natural treatment of the moving shoreline. This is computed as part of the solution, so there is no need for any tracking of the wet/dry front. The model can thus deal with multiple shorelines and splitting of the water mass, as can occur during an overtopping event. The Boussinesq equations are solved using finite differences, as smooth solutions are expected pre-breaking.

A novel feature of the present numerical model is the inclusion of a full model for a piston paddle wavemaker. It allows for complete simulations of shallow water laboratory experiments, including the wave generation process. The problem of the time-varying domain size, due to the paddle movement, is overcome by implementing a domain transformation, so that calculations are performed in a fixed domain. This horizontal transformation bears resemblance to a σ-transformation used in the vertical coordinate in three-dimensional free surface flow problems (as an example see [33]).

The paper is structured as follows. Section 2 outlines the governing equations of the hybrid model and how the piston paddle is modelled as a moving boundary. The treatment of wave breaking is also discussed. Section 3 describes the numerical methods used to solve the governing equations, the wetting and drying algorithm, and the boundary conditions. Special attention is given to the moving grid adjacent to the paddle. The model is validated against analytical solutions and experimental measurements in Sections 4 and 5. In Section 4.1 an exact numerical solution for solitary waves in the present Boussinesq equations is derived, together with the relation between the solitary wave amplitude and celerity.

2. Governing equations

The hybrid model uses two sets of governing equations. The enhanced Boussinesq equations derived by Madsen and Sørensen [24] are used pre-breaking and the nonlinear shallow water equations post-breaking. A stage-discharge (η, q) formulation is adopted for both equation sets, where η denotes the water surface level above a prescribed horizontal datum and

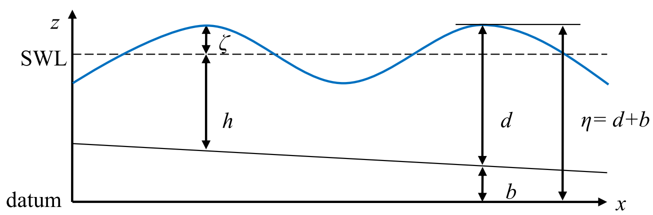

Fig. 1. Definition sketch.