Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Physics 224 (2007) 288–313
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

A Darcy law for the drift velocity in a two-phase flow model

H. Guillard [a,*], F. Duval [b]

[a] INRIA, B.P. 93, 06902 Sophia Antipolis Cedex, France
[b] Institut de Radioprotection et de Sûreté Nucléaire (IRSN), BP3-13115 St. Paul-lez-Durance Cedex, France

Received 4 September 2006; received in revised form 9 February 2007; accepted 12 February 2007
Available online 12 March 2007

Abstract

This work deals with the design and numerical approximation of an Eulerian mixture model for the simulation of two-phase dispersed flows. In contrast to the more classical two-fluid or Drift-flux models, the influence of the velocity disequilibrium is taken into account through dissipative second-order terms characterized by a Darcy law for the relative velocity. As a result, the convective part of the model is always unconditionally hyperbolic. We show that this model corresponds to the first-order equilibrium approximation of classical two-fluid models. A finite volume approximation of this system taking advantage of the hyperbolic nature of the convective part of the model and of the particular structural form of the dissipative part is proposed. Numerical applications are presented to assess the capabilities of the model.

Keywords: Two-phase flows; Bubbly flows; Drift-flux; Riemann solver; Darcy law; Chapman–Enskog expansion

1. Introduction

Two-phase flows appear in a large number of engineering applications including the petroleum industry as well as industrial processes involving bubble column reactors (oxidation, hydrogenation, mixing of heterogeneous compositional materials, etc.). They also take an important place in the nuclear industry for reactors in operating conditions as well as in severe accident conditions. The macroscopic description of such flows is usually obtained by some averaging procedure [1–3], and is performed essentially by two large classes of models.

The first class that corresponds to the most general form obtained from the averaging procedure consists of separate mass, momentum and energy conservation equations written for each phase of the multiphase system. This corresponds to the well-known two-fluid model characterized in their most general form by two different velocities and pressures for each phase and possibly supplemented by topological equations [4,5] as well as the more classical one pressure, two velocity models whose closure is realized by the assumption of equality of the phase pressures [6–9].

* Corresponding author. Tel.: +33 04 92 38 77 96; fax: +33 04 92 38 79 80.
E-mail address: Herve.Guillard@sophia.inria.fr (H. Guillard).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2007.02.025

The second class of model corresponds to the mixture models and consists of single conservation equation for the mixture (mass, momentum and energy) and a mass conservation equation for one of the two phases. In this class, one can distinguish the so-called drift-flux models [10–13] for which an additional relation for the relative velocity is provided and the homogeneous models [14] that assumes a no-slip hydrodynamic law between the two phases. Some intermediate models have been proposed between these two large classes and we refer to [15,16] for some examples and to [3] for a presentation and a discussion of these models.

The two-fluid models are widely used for two-phase flow studies but have some drawbacks. The first difficulty concerns the modeling of interphase exchange terms. Indeed, there is a lot of practical situations for which the use of a two-fluid model requires an accurate modeling of the interfacial transfer terms (drag, lift, added mass, etc.) in order to capture correctly the two-phase flow behavior. Furthermore, from a mathematical point of view, it is well known that the one pressure two-fluid system is not unconditionally hyperbolic depending on the closure for interfacial terms. Finally from a numerical point of view, the complexity of the modeling, the presence of non-conservative products as well as the possible loss of hyperbolicity of the models make the approximation of these models difficult and dubious.

The mixture models are simpler and are expected to be well suited for situations where the two phases are well coupled. However, the mixture models also experience some difficulties. The homogeneous models are unable to take into account even a slight disequilibrium between the velocities of the phases and on the other hand, the drift-flux models have also some drawbacks. These drawbacks lie mainly in the constitutive relation for the drift velocity. First, the resulting system is not always hyperbolic. For instance, for two phases having different densities, both the Stokes and the Churn drift correlations leads to non-hyperbolic systems. The second weak point of this type of model is that the complexity of the drift-flux correlation sometimes prevents the existence of an analytical expression of the fluxes in terms of conservative variables and thus the approximation of these models requires complex and specialized methods [12,13].

In this paper, we examine situations where the two-phase flows are characterized by a stiff mechanical relaxation. By stiff mechanical relaxation, we mean a situation where the velocities of the two-phase tends towards a common value under the effects of the drag forces. In these situation, the use of homogeneous model is not always successful as even a slight velocity disequilibrium can have a large influence on the behavior of the system and it is often more accurate to be able to retain some non-equilibrium effects. We will show that this is possible in the framework of mixture model by deriving the first-order Chapman–Enskog asymptotic system corresponding to two-fluid models. In particular, we will show that the use of the Chapman–Enskog expansion allows to express the relative velocity between the phases by a Darcy law. Moreover, the resulting model is unconditionally hyperbolic and dissipative. This type of model offers therefore some advantages over the more classical two-fluid or drift-flux models. This paper is organized as follows: In the first section we recall in an isothermal setting the classical two-fluid model and show how to derive from it a reduced mixture model characterized by a Darcy law for the drift (relative) velocity. The second section is devoted to the mathematical analysis of this model. We will see that the convective part of the model is unconditionnaly hyperbolic in contrast to the more classical two-fluid or drift-flux model but also that it is dissipative and consistent. In the third section of this paper, we describe the numerical approximation of this system that we have used while the last section presents some numerical experiments.

2. A Darcy law for the drift velocity

2.1. The two-fluid multiphase model

We consider the well-known one pressure two-fluid model. Restricted to isothermal flows this model consists in separate mass and momentum conservation equations for each phase:

$$\frac{\partial \alpha_1 \rho_1}{\partial t} + \nabla \cdot (\alpha_1 \rho_1 \mathbf{v}_1) = 0, \tag{1}$$

$$\frac{\partial \alpha_1 \rho_1 \mathbf{v}_1}{\partial t} + \nabla \cdot (\alpha_1 \rho_1 \mathbf{v}_1 \otimes \mathbf{v}_1) + \nabla(\alpha_1 p) = p_I \nabla \alpha_1 + \mathbf{F}^d + \alpha_1 \rho_1 \mathbf{g}, \tag{2}$$