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A comparison of vortex and pseudo-spectral methods for the simulation of periodic vortical flows at high Reynolds numbers

Wim M. van Rees [a], Anthony Leonard [b], D.I. Pullin [b], Petros Koumoutsakos [a,*]

[a] *Chair of Computational Science, ETH Zürich, CH-8092, Switzerland*
[b] *Graduate Aeronautical Laboratories, California Institute of Technology, Pasadena, CA 91125, USA*

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ABSTRACT

We present a validation study for the hybrid particle-mesh vortex method against a pseudo-spectral method for the Taylor–Green vortex at $Re_\Gamma = 1600$ as well as in the collision of two antiparallel vortex tubes at $Re_\Gamma = 10{,}000$. In this study we present diagnostics such as energy spectra and enstrophy as computed by both methods as well as point-wise comparisons of the vorticity field. Using a fourth order accurate kernel for interpolation between the particles and the mesh, the results of the hybrid vortex method and of the pseudo-spectral method agree well in both flow cases. For the Taylor–Green vortex, the vorticity contours computed by both methods around the time of the energy dissipation peak overlap. The energy spectrum shows that only the smallest length scales in the flow are not captured by the vortex method.

In the second flow case, where we compute the collision of two anti-parallel vortex tubes at Reynolds number 10,000, the vortex method results and the pseudo-spectral method results are in very good agreement up to and including the first reconnection of the tubes. The maximum error in the effective viscosity is about 2.5% for the vortex method and about 1% for the pseudo-spectral method. At later times the flows computed with the different methods show the same qualitative features, but the quantitative agreement on vortical structures is lost.

1. Background

Vortex methods are arguably the first numerical method used for the simulation of vortical flows starting with the hand-calculations of Rosenhead in the beginning of last century [1]. Vortex methods were considered as the method of choice for external flows with compact vorticity [2] due to their low numerical dissipation and they were among the first techniques used for simulations of 3D vortical flows [3–5]. In recent years it was realized [6] that the accuracy of the method hinges on the use of a regularisation procedure to remedy the inaccuracies due to the distortion of the computational elements which follows from their Lagrangian adaptivity. In the remeshed vortex method (rVM) [7–9], Lagrangian vortex particles are used to simulate the convective part of the equations and particles are mapped onto grid nodes at each time step so as to ensure the convergence of the method and to compute efficiently the solution of the Poisson equation that determines their velocity. This gives the rVM some inherent advantages over other methods, such as its adaptivity and the lack of a CFL restriction on the timestep, which allows large timesteps during the simulation. It is important to note that the use of a grid based solver for the Poisson equation accommodates a wide range of boundary conditions that may not be possible when using tech-

* Corresponding author. Tel.: +41 44 632 52 58; fax: +41 44 632 17 03.
E-mail address: petros@ethz.ch (P. Koumoutsakos).

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niques such as the Fast Multipole Method [10]. We are interesting in exploiting these advantages for high Reynolds number (Re_Γ = 10,000) vortical flows while retaining the accuracy of the results. The method has already successfully been used to perform direct numerical simulations up to Re_Γ = 7500 [11,12]. To this end, we will compare our remeshed vortex method with a pseudo-spectral method. The pseudo-spectral method is well suited for the simulation of high Reynolds number flows in simple domains and can be considered as a reference method. The goal of the current study is to validate the vortex method as an accurate and fast alternative to the pseudo-spectral method for high Reynolds number flow cases.

The first study of comparing the vortex method with the pseudo-spectral method was undertaken by Cottet et al. [13]. In that study, the comparison was performed for isotropic turbulence in a periodic box at initial $Re_\lambda \approx 100$, and for the reconnection of two vortex tubes at Re_Γ = 3500. In the last case the flow was still laminar. For both cases it was found that the vortex method resolves the large- and medium scales in the flow well. In addition, the simulation of isotropic turbulence showed that the vortex method does not suffer from accumulation of energy in the tail of the energy spectrum, whereas the spectral method does. Furthermore, an underresolved flow simulation of the colliding vortex tubes showed that the pseudo-spectral method generates spurious vortex structures, but with the vortex method the large scales are still adequately resolved and no spurious vorticity appears. In a recent work by Cocle et al. [14] a vortex method and a pseudo-spectral code are used to compare various multiscale subgrid models in LES of homogeneous isotropic turbulence. They report little difference between the two methods in the obtained spectra when using the same subgrid model.

In this study we focus on the accuracy of the vortex method at higher Reynolds numbers than in [13], and we study the effect of employing a higher order remeshing kernel in the remeshed vortex method. The paper is organized as follows. First we describe the methods used in this study. Then we report on the simulation results for a Taylor–Green vortex at Re_Γ = 1600. Finally we move on to a flow case at Re_Γ = 10,000 and describe the comparison between the vortex method results and the pseudo-spectral method results.

2. Governing equations and numerical method

2.1. Vortex method

The evolution of viscous incompressible flow is considered as described by the Navier–Stokes equations in Lagrangian vorticity form:

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \nu\Delta\boldsymbol{\omega}, \tag{1}$$

and

$$\Delta\boldsymbol{\Psi} = \nabla \times \mathbf{u} = -\boldsymbol{\omega}, \tag{2}$$

where $\boldsymbol{\Psi}$ is the vector streamfunction.

The equations are discretized using a remeshed vortex method (rVM). In the traditional vortex particle method, the vorticity field is approximated using particles:

$$\boldsymbol{\omega}(\mathbf{x}, t) \approx \sum_p \boldsymbol{\Gamma}_p(t)\zeta_\epsilon(\mathbf{x} - \mathbf{x}_p(t)), \tag{3}$$

where $\boldsymbol{\Gamma}_p(t)$ and $\mathbf{x}_p(t)$ denote the particle strength and particle position, respectively, of the pth particle at time t. In our hybrid formulation of the vortex particle method, the kernel function ζ_ϵ is used for interpolation between the particles and the grid (see the next subsection). To compute the Fourier-transform of our quantities, we assume a Fourier interpolation on the grid rather than using Eq. (3).

Discretizing the Navier–Stokes equations with particles results in a set of ordinary differential equations (ODEs) for the particle strengths and the particle positions:

$$\frac{d\boldsymbol{\Gamma}_p}{dt} = \upsilon_p(\boldsymbol{\omega} \cdot \nabla^h)\mathbf{u} + \nu\Delta^h\boldsymbol{\Gamma}_p, \tag{4}$$

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p, t). \tag{5}$$

Here $\upsilon_p = h^3$ are the particle volumes. These differential equations are integrated in time using either a low-storage third order Runge–Kutta method [15] (RK3), or a fourth order Runge–Kutta method (RK4). The discretized operator for the viscous term is evaluated with a centered fourth-order finite difference scheme. The stretching term is rewritten in its transpose formulation $\omega_k\partial u_k/\partial x_i$, and is discretized with fourth order finite differences. Every timestep the particles are remeshed onto a uniform Cartesian grid to enforce that the particles always overlap. In this way the occurrence of spurious vortical structures is prevented and convergence of the method is ensured [16,7]. The velocities are computed from the vorticity by solving Eq. (2) in Fourier space on the grid. This ensures that the velocity field is spectrally divergence-free. To ensure a divergence-free vorticity field ($\nabla \cdot \omega = 0$), a solenoidal reprojection based on the Helmholtz decomposition of the vorticity field is done in spectral space every 5–10 timesteps.