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Short Note

Blending technique for compressible inflow turbulence: Algorithm localization and accuracy assessment

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1. Introduction

The accuracy of large eddy and direct numerical simulations (LES and DNS) of spatially developing flows is dependent on the physical realism of the inflow turbulence. There are many possible ways to generate the inflow turbulence, with varying degrees of physical realism and applicability.

The methods by Batten et al. [1] and Di Mare et al. [2] are two examples of rather generally applicable methods to generate "synthetic turbulence", where the mean velocity profile, the Reynolds stress tensor, and the energy spectrum can be arbitrarily prescribed. Keating et al. [3] provided a comprehensive review of different methods, and tested some on spatially developing channel flow. They found that the lack of phase information in the synthetic turbulence at the inflow caused a development region before the resolved turbulence was accurate. The recycling technique by Lund et al. [4] is commonly used for spatially developing boundary layers. Instantaneous turbulence from within the domain is rescaled and recycled at the inflow, resulting in more realistic phase information. The method has been extended to compressible flows, with a comparative assessment given by Xu and Martin [5].

Several important problems in fluid mechanics have inflows with uniform mean velocity and isotropic turbulence; some examples include shock/turbulence interaction and by-pass transition with leading-edge effects taken into account [6]. To avoid the development region resulting from synthetic turbulence methods, one can instead pre-compute a database of isotropic turbulence to the desired state that is then convected into the domain using Taylor's hypothesis. There are two potential problems with this approach. First, Lee et al. [7] showed that Taylor's hypothesis is only valid for the hydrodynamic parts of the flow field, like vorticity and kinetic energy, but not for the acoustic part. Thus care is needed for problems where the acoustics are of primary importance. However, one should note that other inflow techniques likely

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suffer from this problem as well. The second potential problem, and the more prohibiting one, is that the cost of pre-computing the inflow database becomes exceedingly high if a long record in time in needed. For example, statistical convergence in by-pass transition may require an order of magnitude longer record of inflow turbulence than the domain flow-through time of the calculation [6].

One particulary appealing solution to this problem was proposed by Xiong et al. [6], who suggested that several independent realizations (or snap-shots) of isotropic turbulence be blended together along the streamwise direction to create an arbitrarily long database. Computing several smaller cases instead of one large requires much less memory, which is often the limiting factor in modern high-performance computing. In addition, the isotropy of the snap-shots can be utilized (by rotating them) to create a very long database using only a limited number of snap-shots [6]. Xiong et al. showed how to blend the velocity components such that the second-order pointwise moments are preserved, and estimated the error in the two-point correlations. Finally, they showed how the blending introduces an error in the dilatation field, and suggested that this be removed through solution of a Poisson equation. They then showed a qualitative 'proof-of-concept' without detailed assessment of the accuracy of the method. Thus the first objective of the present work is to quantitatively assess the accuracy of the blending technique. This is done by considering spatially decaying turbulence, and comparing the results both to temporally decaying turbulence as well as cases with synthetic turbulence at the inlet.

In addition, we present a simple modification of the Xiong et al. method that makes it more amenable to large-scale computing. They originally proposed to solve the Poisson equation in the full domain of the database, implying that the memory required to blend N_f realizations of size N^3 scales as $N_f \times N^3$. Therefore, for long inflow databases, memory limitations alone may dictate that more processors are used to create the database than are used for the actual flow calculation. In the present work we approach the problem differently: by 'localizing' the Poisson system for dilatation removal, each blending region becomes independent of every other. This allows us to either perform the N_f blending operations sequentially, with memory-usage scaling as N^3 (with a lower constant of proportionality as well), or to perform the N_f blending operations in so-called 'embarrasingly parallel' fashion. It also enables additional realizations to be added to an existing database as they are required, including while the main simulation is running, which may be useful when the time required for statistical convergence is not known *a priori*.

2. Blending procedure

Consider the concatenation of two independent but statistically identical periodic boxes of turbulence of size $[0, 2\pi)^3$ into a larger box $[-2\pi, 2\pi) \times [0, 2\pi)^2$. Following [6], the original velocity fields $u_i^{(1)}$ and $u_i^{(2)}$ (with zero mean) are blended as

$$u_i = u_i^{(1)} \cos\alpha + u_i^{(2)} \sin\alpha - \partial_i \varphi, \quad |x_1| < l_b \tag{1}$$

where α is varied smoothly over the blending region of size l_b and $\partial_i \varphi$ will be used later to remove the erroneous dilatation. One choice for α is

$$\alpha = \frac{\pi\beta}{2}, \quad \beta = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi x_1}{2 l_b}\right), \quad |x_1| < l_b$$

for which we note that $d\alpha/dx_1 \leqslant \pi^2/(8 l_b)$. Using (1) with neglected $\partial_i \varphi$, Xiong et al. [6] showed that the two-point correlation

$$R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x}) u_j(\mathbf{x}+\mathbf{r})\rangle \approx \left[1 - \frac{1}{2}\left(r_1 \frac{d\alpha}{dx_1}\right)^2\right] R_{ij}^{(1)}(\mathbf{r})$$

where $R_{ij}^{(1)} = R_{ij}^{(2)}$ is the correlation function of the original fields. This result makes use of a Taylor expansion of $\cos\alpha$ and the fact that the fields are independent realizations. It shows that the blended field retains the second-order single-point statistics of the original fields as well as the transverse two-point correlations, while the streamwise two-point correlation is lowered by an amount controlled by the size of the blending region l_b (through the bound on $d\alpha/dx_1$).

In addition, the gradients of the blended field are altered in a similar way, as

$$\partial_j u_i = \cos(\alpha)\partial_j u_i^{(1)} + \sin(\alpha)\partial_j u_i^{(2)} + \left(-u_i^{(1)} \sin\alpha + u_i^{(2)} \cos\alpha\right)\frac{d\alpha}{dx_1}\delta_{1j} - \partial_{ij}^2 \varphi \tag{2}$$

where the third term is the error due to the blending. To get a sense for this error, consider the profiles in Fig. 1.

While the blended vorticity reasonably represents the original fields, the blended dilatation has a huge peak inside the blending region. Xiong et al. [6] suggested removing the erroneous dilatation by obtaining φ from the Poisson equation

$$\partial_{jj}^2 \varphi = q = (-u_1^{(1)} \sin\alpha + u_1^{(2)} \cos\alpha)\frac{d\alpha}{dx_1} \tag{3}$$

with Neumann and periodic boundary conditions in the streamwise and transverse directions, respectively. They therefore solved Eq. (3) in the domain of the full database $(N_f 2\pi) \times (2\pi)^2$, leading to memory-usage that increases with the length of the database. This method effectively reduces the dilatation error to within the variation of the original fields, as seen in Fig. 1.