Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Physics 220 (2006) 109–138
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

An immersed interface method for viscous incompressible flows involving rigid and flexible boundaries

D.V. Le [a], B.C. Khoo [a,b,*], J. Peraire [a,c]

[a] Singapore-MIT Alliance, 4 Engineering Drive 3, National University of Singapore, Singapore 117576, Singapore
[b] Department of Mechanical Engineering, National University of Singapore, Kent Ridge Crescent, Singapore 119260, Singapore
[c] Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, MA 02139, USA

Received 22 September 2005; received in revised form 5 May 2006; accepted 5 May 2006
Available online 13 July 2006

Abstract

We present an immersed interface method for the incompressible Navier–Stokes equations capable of handling both rigid and flexible boundaries. The immersed boundaries are represented by a number of Lagrangian control points. In order to ensure that the no-slip condition on the rigid boundary is satisfied, singular forces are applied on the fluid. The forces are related to the jumps in pressure and the jumps in the derivatives of both pressure and velocity, and are interpolated using cubic splines. The strength of the singular forces at the rigid boundary is determined by solving a small system of equations at each timestep. For flexible boundaries, the forces that the boundary exerts on the fluid are computed from the constitutive relation of the flexible boundary and are applied to the fluid through the jump conditions. The position of the flexible boundary is updated implicitly using a quasi-Newton method (BFGS) within each timestep. The Navier–Stokes equations are discretized on a staggered Cartesian grid by a second order accurate projection method for pressure and velocity and the overall scheme is second order accurate.

Keywords: Immersed interface method; Navier–Stokes equations; Cartesian grid method; Finite difference; Fast Poisson solvers; Irregular domains; Fluid–membrane interaction

1. Introduction

In this paper, we present a numerical method for solving viscous, incompressible flow problems involving both moving interfaces and rigid boundaries. One of the challenges in these problems is that the fluid motion, the flexible interface motion and the interaction with the immersed rigid boundaries must be computed simultaneously. This is necessary in order to account for the complex interaction between the fluid and the immersed boundaries. An example of interface problems that we consider is shown in Fig. 1. In a 2-dimensional bounded

* Corresponding author. Address: Department of Mechanical Engineering, National University of Singapore, Kent Ridge Crescent, Singapore 119260, Singapore. Tel.: +65 68742889; fax: +65 67791459.
E-mail address: mpekbc@nus.edu.sg (B.C. Khoo).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2006.05.004

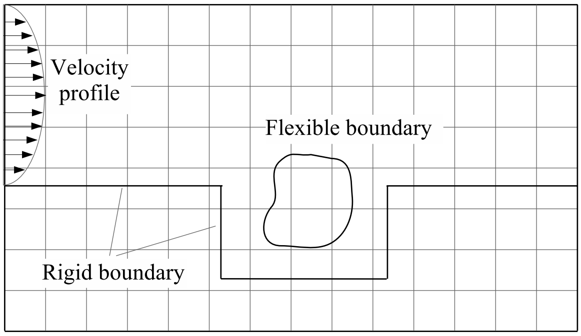

Fig. 1. A typical domain in which the Navier–Stokes equations are solved. The flexible interface and the rigid boundary are immersed in a uniform Cartesian grid.

domain Ω that contains a material interface $\Gamma(t)$, we consider the incompressible Navier–Stokes equations, written as

$$\rho(\boldsymbol{u}_t + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}) + \nabla p = \mu \Delta \boldsymbol{u} + \boldsymbol{F}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

with boundary conditions

$$\boldsymbol{u}|_{\partial\Omega} = \boldsymbol{u}_b, \tag{3}$$

where $\boldsymbol{u}$ is the fluid velocity, p is the pressure, ρ is the density, and μ the viscosity of the fluid. Throughout this paper, we assume that the fluid density ρ and the viscosity μ are constant over the whole domain. The effect of the material interface $\Gamma(t)$ immersed in the fluid results in a singular force $\boldsymbol{F}$ which has the form

$$\boldsymbol{F}(\boldsymbol{x}, t) = \int_{\Gamma(t)} \boldsymbol{f}(s, t)\delta(\boldsymbol{x} - \boldsymbol{X}(s, t))\,\mathrm{d}s, \tag{4}$$

where $\boldsymbol{X}(s,t)$ is the arc-length parametrization of $\Gamma(t)$, s is the arc-length, $\boldsymbol{x} = (x, y)$ is spatial position, and $\boldsymbol{f}(s,t)$ is the force strength. Here, $\delta(\boldsymbol{x})$ is the two-dimensional Dirac function. The motion of the interfaces satisfies

$$\frac{\partial}{\partial t}\boldsymbol{X}(s, t) = \boldsymbol{u}(\boldsymbol{X}, t) = \int_{\Omega} \boldsymbol{u}(\boldsymbol{x}, t)\delta(\boldsymbol{x} - \boldsymbol{X}(s, t))\,\mathrm{d}\boldsymbol{x}. \tag{5}$$

In our proposed numerical method, the Navier–Stokes equations are discretized using a standard finite difference method on a staggered Cartesian grid. Methods utilizing a Cartesian grid for solving interface problems or problems with complex geometry have become popular in recent years. Existing Cartesian grid methods for interface problems can be categorized into two general groups: methods that determine the jump conditions across the interface and incorporate them into the finite difference scheme and methods that smooth out the singular force before it is applied to the fluid. Our method which is based on the immersed interface method originally proposed by LeVeque and Li [20,21] falls into the first group. The immersed boundary method introduced by Peskin [26] belongs to the second group.

Peskin's immersed boundary method has proven to be a very useful method for modelling fluid-structure interaction involving large geometry variations. This method has been applied to many biological problems involving flexible boundaries [10,11,25,34]. In the immersed boundary method, the force densities are computed at the control points which are used to represent the boundaries. The force densities are then spread to the Cartesian grid points by a discrete representation of the delta function. The Navier–Stokes equations with the forcing terms are then solved for pressure and velocity at the Cartesian grid points. Further details on the immersed boundary method can be found in [26] and the references therein. The immersed boundary method has several attractive features: the method is simple to implement, it can handle complex geometries easily and it uses standard regular Cartesian grid Navier–Stokes solvers. However, since the immersed boundary method uses the discrete delta function approach, it smears out sharp interface to a thickness of