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A bandwidth-optimized WENO scheme for the effective direct numerical simulation of compressible turbulence

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Abstract

Two new formulations of a symmetric WENO method for the direct numerical simulation of compressible turbulence are presented. The schemes are designed to maximize order of accuracy and bandwidth, while minimizing dissipation. The formulations and the corresponding coefficients are introduced. Numerical solutions to canonical flow problems are used to determine the dissipation and bandwidth properties of the numerical schemes. In addition, the suitability and accuracy of the bandwidth-optimized schemes for direct numerical simulations of turbulent flows is assessed in decaying isotropic turbulence and supersonic turbulent boundary layers.

Keywords: Direct numerical simulation; Shock capturing; Bandwidth optimization; Turbulence; Compressible flow

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1. Introduction

The detailed simulation of compressible turbulent flows requires solving the conservation of mass, momentum and energy equations. For direct numerical simulations (DNS) all possible turbulent length scales and time scales must be resolved by the numerical method. Thus, DNS requires accurate representation of time-dependent propagation of high wavenumber (or high frequency), small amplitude waves. In addition, compressible turbulent flows are characterized by shockwaves that result in a sudden change of the fluid properties. Therefore, methods for compressible turbulent flows require robust shock capturing, as well as minimal dissipation and dispersion errors.

Resolving the shock thickness is impractical for the detailed simulation of turbulence, as the mean free path is typically orders of magnitude smaller than that of the Kolmogorov length scale [1,2]. In the present simulations, we resolve all turbulent length scales and time scales, while shocks are not being resolved. In turn, the simulations within are "effective" direct numerical simulations.

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Fig. 1 shows a numerical schlieren image from a DNS of a shock and turbulent boundary layer interaction at Mach 3 and Reynolds number based on momentum thickness $Re_\theta = 2400$ [3,4]. In this flow, a turbulent boundary layer is convected over a 24° compression ramp, which generates a shockwave. The turbulent structures in the incoming boundary layer are apparent. The unsteady nature of the boundary layer causes the shock to wrinkle and oscillate near the corner. Downstream of the corner, the boundary layer remains compressed between the wall and the compression-corner shock, and additional shocks emanate from the large structures in the wake of the boundary layer and merge with the compression-corner shock while convecting downstream. Computing all the relevant turbulence structures in this flow requires high order of accuracy and bandwidth-resolving efficiency, as well as shock capturing for the unsteady shockwaves. Weighted essentially non-oscillatory (WENO) schemes provide a means for the DNS of compressible turbulent flow.

In WENO schemes [5], the numerical flux is computed as the weighted sum of a set of candidate flux approximations. The weights depend dynamically on the local smoothness of the data. Smoothness measurements cause stencils that span large flow field gradients to be assigned small relative weights; any candidate stencil containing a shock receives a nearly zero weight. In completely smooth regions, weights revert to optimal values, where optimal is defined by, e.g. maximum order of accuracy or maximum bandwidth. This weighting procedure makes the WENO schemes more robust than their predecessors, the essentially non-oscillatory (ENO) schemes (see for example [6,7]), which use the single smoothest candidate stencil to the exclusion of the others.

Jiang and Shu [8] cast WENO into finite-difference form and provide an efficient numerical implementation of the shock-capturing technique so that conditional statements are avoided. This scheme, which is referred as WENO-JS hereafter, provides robust shock-capturing, high-order accuracy and efficient implementation on distributed memory, multiprocessor machines. However, WENO-JS is too dissipative for the detailed simulation of turbulent flow.

There have been a number of efforts to overcome the deficiencies of conventional shock-capturing schemes for the detailed simulation of compressible turbulence. Adams and Shariff [9] developed a hybrid scheme that couples compact upwind and shock-capturing ENO schemes, in which the ENO scheme is activated only around discontinuities. Pirozzoli [10] follows a similar approach and replaces the non-conservative formulation of the compact scheme with a conservative one and the ENO with WENO, resulting in a more stable and accurate algorithm. Ren et al. [11] improve the hybrid compact-WENO scheme [10] by removing the abrupt switch between the compact and the shock-capturing schemes through the use of a weighted average of the two schemes. Hill and Pullin [12] use a version in which the non-shock-capturing scheme is centered rather than upwind-biased, thereby reducing the overall dissipation in smooth flow regions. Kim and Kwon [13] propose an additional formulation of a hybrid central-difference WENO scheme with an alternative weighting function for the two schemes.

Rather than combining two different schemes and choosing when to use each one, in this paper we apply a modified version of Jiang and Shu's method everywhere in the flowfield. The purpose of the non-linear adaptation mechanism in WENO schemes is choosing when to use the optimal stencil and when to use a smaller stencil to avoid interpolation across high gradients (which result in numerical oscillations). Thus, there are two sources of WENO dissipation: that associated with the adaptation mechanism and that of the optimal stencil.

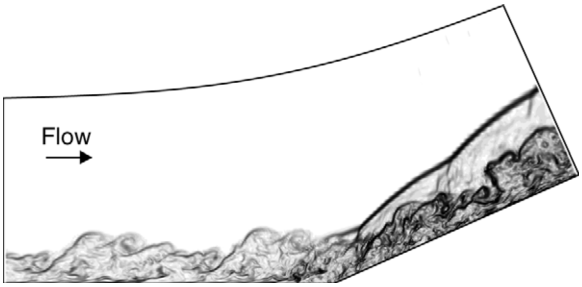

Fig. 1. Numerical schlieren from the DNS of a shockwave/turbulent boundary layer interaction. The incoming boundary layer is at Mach 3 and $Re_\theta = 2400$ [3,4].