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The force/work differencing of exceptional points in the discrete, compatible formulation of Lagrangian hydrodynamics

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Abstract

This study presents the force and mass discretization of exceptional points in the compatible formulation of Lagrangian hydrodynamics. It concludes a series of papers that develop various aspects of the theoretical exposition and the operational implementation of this numerical algorithm. Exceptional points are grid points at the termination of lines internal to the computational domain, and where boundary conditions are therefore not applied. These points occur naturally in most applications in order to ameliorate spatial grid anisotropy, and the consequent timestep reduction, that will otherwise arise for grids with highly tapered regions or a center of convergence. They have their velocity enslaved to that of neighboring points in order to prevent large excursions of the numerical solution about them. How this problem is treated is given herein for the aforementioned numerical algorithm such that its salient conservation properties are retained. In doing so the subtle aspects of this algorithm that are due to the interleaving of spatial contours that occur with the use of a spatially-staggered-grid mesh are illuminated. These contours are utilized to define both forces and the work done by them, and are the central construct of this type of finite-volume differencing. Additionally, difficulties that occur due to uncertainties in the specification of the artificial viscosity are explored, and point to the need for further research in this area.

Keywords: Lagrangian; Hydrodynamics; Energy conserving; High speed flow; Artificial viscosity

1. Introduction

Most physical applications that involve Lagrangian or ALE hydrodynamics calculations employ grids that must be unstructured to some degree to avoid the spatial grid stiffness that would otherwise occur, resulting in an unacceptable decrease in timestep. While the problem of differencing the hydro equations about exceptional, or irregular, points of such grids has been addressed in a previous paper for the case where the exact one-dimensional symmetry limit is desired in curvilinear coordinates [1], the solution presented requires rotations of the force that may not be desirable for general cases, particularly those where low-dimensional symmetry is not important and does not attain to any approximate degree in the solution. However, the general

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discussion given therein of the timestep vicissitudes of such grids remains valid and is not repeated here. An example of the type of grid previously considered is shown in Fig. 1. Here the terminated lines, and associated exceptional points, are displayed as hollow circles that are placed at the midpoints of straight lines connecting neighboring regular points. This is opposed to being placed on a common radius as previously depicted in [1]. It is in general necessary to enslave the terminated points shown in Fig. 1 to prevent large unphysical perturbations in the solution from occurring about them when disturbances such as shock waves propagate in any direction across them. Thus the force and mass discretizations that are suitable for regular zones that do not contain these points must be appropriately modified where they are present in the grid. This requires a careful investigation of the discretization properties of the underlying hydro algorithm that elucidates all assumptions, transparent or hidden. To this end, Section 2 gives a brief review of the hydro algorithm that forms the title of this paper. This is, however, a new and concise presentation that displays essential features that complement previous expositions [2,3].

In Section 3 the types of grids encountered are briefly detailed as well as the basic constraints that are enforced when both mass and force from the exceptional points are "donated" to neighbors to which their velocity is enslaved, making the former "nondynamical". Section 4 gives numerical results that validate the discretization rules introduced in Section 3, and quantifies the magnitude of errors that necessarily occur when strong shock waves encounter these points. In particular, special attention is paid to the artificial viscosity forces as they are velocity dependent; and since velocity interpolation is utilized, these forces can result in sensitivities that are difficult to counter by any general prescriptions. Last, a discussion of this work and its principal conclusions is given.

2. Discrete, compatible Lagrangian hydrodynamics

The discrete, compatible formulation of Lagrangian hydrodynamics [3] essentially modernizes older forms of Lagrangian hydro [4]. It places this type of numerical algorithm into a simple and consistent framework where conservation of total energy plays the central role, but where the principal dependent variables remain density, velocity, and specific internal energy. Like all the older versions of Lagrangian hydrodynamics it employs a staggered grid in space with velocity and position carried on points "p", and density, specific internal energy, and stress centered in zones "z". However, both zones and points are considered to be surrounded by interleaved volumes circumscribed by lines in 2D (or surfaces in 3D) that are termed the "coordinate-line" and "median" meshes, respectively, as shown in Fig. 2. This interleaved topology allows for a simple finite-volume calculation of forces. These act from zones that carry a mass M_z, and onto points that carry a mass M_p, where "z" and "p" are integer indices that range over all zones and points, respectively. Auxiliary quantities denoted as "corner" masses and forces are introduced; these are unique and common to both a zone and a given point of that zone, and thus carry both the zone and the point indices. The zone and point masses and the total force acting on a point are then constructed from these more primitive entities as simple sums. The corner mass is denoted as m_z^p or m_p^z, and the corner force as $\vec{f}_z^{\,p}$ or $\vec{f}_p^{\,z}$, where the lower index denotes that which

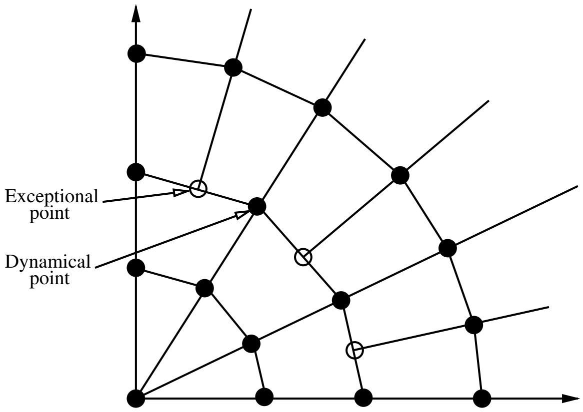

Fig. 1. Typical mesh with exceptional points. Exceptional points are enslaved to adjacent dynamical points on an angular grid line.