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A hybrid Godunov method for radiation hydrodynamics

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ABSTRACT

From a mathematical perspective, radiation hydrodynamics can be thought of as a system of hyperbolic balance laws with dual multiscale behavior (multiscale behavior associated with the hyperbolic wave speeds as well as multiscale behavior associated with source term relaxation). With this outlook in mind, this paper presents a hybrid Godunov method for one-dimensional radiation hydrodynamics that is uniformly well behaved from the photon free streaming (hyperbolic) limit through the weak equilibrium diffusion (parabolic) limit and to the strong equilibrium diffusion (hyperbolic) limit. Moreover, one finds that the technique preserves certain asymptotic limits. The method incorporates a backward Euler upwinding scheme for the radiation energy density E_r and flux F_r as well as a modified Godunov scheme for the material density ρ, momentum density m, and energy density E.

The backward Euler upwinding scheme is first-order accurate and uses an implicit HLLE flux function to temporally advance the radiation components according to the material flow scale. The modified Godunov scheme is second-order accurate and directly couples stiff source term effects to the hyperbolic structure of the system of balance laws. This Godunov technique is composed of a predictor step that is based on Duhamel's principle and a corrector step that is based on Picard iteration. The Godunov scheme is explicit on the material flow scale but is unsplit and fully couples matter and radiation without invoking a diffusion-type approximation for radiation hydrodynamics. This technique derives from earlier work by Miniati and Colella (2007) [41]. Numerical tests demonstrate that the method is stable, robust, and accurate across various parameter regimes.

1. Introduction

Radiation hydrodynamics is a dynamical description of fluid material interacting with electromagnetic radiation and is appropriate whenever radiation governs the transport of energy and momentum in the fluid. Many phenomena in plasma physics and astrophysics are governed by radiation hydrodynamics, some examples include: star formation, supernovae, accretion disks, radiatively driven outflows, stellar convection, and inertial confinement fusion [5,37,40]. In these applications, the radiation field heterogeneously couples to the material dynamics such that radiative effects are strong in some parts of the system and weak in other parts. These variations give rise to characteristically different dynamical properties (i.e., advection versus diffusion behavior). The primary objective for developing the numerical technique presented in this paper is to have a computational tool that accurately solves radiation hydrodynamical problems across a range of asymptotic limits. The new algorithmic ideas are cast in such a way that they seem familiar with respect to classical Godunov schemes and can be implemented in existing codes with minimal computational overhead. A future research endeavor is to combine

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the hybrid Godunov method for radiation hydrodynamics with an existing code for MHD (magnetohydrodynamics) such as Athena [52] and investigate full radiation MHD in multiple spatial dimensions.

Some of the initial research in developing numerical methods to solve radiation hydrodynamical problems was carried out by Castor [4], Pomraning [44], Levermore and Pomraning [29], Mihalas and Klein [38], and Mihalas and Weibel-Mihalas [40]. One of the simplest and most successful approaches used in astrophysics was the Zeus code of Stone et al. [53], which relied on operator splitting and a Crank–Nicholson temporal finite difference scheme. Since the introduction of that code, finite volume schemes (e.g. Godunov-type methods) have emerged as a powerful technique for solving hyperbolic conservation laws (i.e., the mathematical framework describing radiation hydrodynamics) [27,28]. Moreover, this integral formulation allows one to more naturally treat boundary conditions, capture shock waves and other discontinuous behavior, investigate complex geometries and multi-dimensions, and implement adaptive mesh refinement [59]. Despite these advantages, there have been significant difficulties in developing a Godunov method to accurately represent radiation hydrodynamical behavior across a range of asymptotic limits. Earlier attempts to construct a Godunov-type technique either (i) neglected the heterogeneity of the matter-radiation coupling and solved the system of equations in a specific limit [9,10], (ii) were based on a model system [3,23], (iii) invoked artificial coupling terms that were based on the reference frame in which the problem was solved [1] as pointed out by Lowrie and Morel [31], or (iv) used a variation of flux limited diffusion [22,25,29,58]. Lowrie and Morel [31] was even critical of the likelihood of developing such a method for full radiation hydrodynamics.

Developing a Godunov method for radiation hydrodynamics has been difficult because numerical difficulties arise from (i) the reference frame one chooses for taking moments of the photon transport equation, (ii) multiscale waves, (iii) stiff source terms, and (iv) solving a hierarchy of radiation transport moment equations to compute the variable tensor Eddington factor f. The first difficulty occurs because one takes moments of the photon transport equation in order to define the radiation quantities that interact with the material components of the system. One encounters problems with the radiation field's specific intensity function $I(\nu,\mathbf{n})$ which is governed by frame-dependent quantities, the radiation frequency ν and directional vector $\mathbf{n}$. Here, one either casts the photon transport equation into the comoving frame (at rest with respect to the local fluid velocity) and contends with complicated transport operators but simple interaction terms $S(\nu,\mathbf{n})$; or one casts the photon transport equation into the Eulerian frame (at rest with respect to the system as a whole) and contends with simple transport operators but complicated interaction terms. The specific intensity is written below in the Eulerian frame, where c is the speed of light and t is time:

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\mathbf{n}\cdot\nabla\right)I(\nu,\mathbf{n})=S(\nu,\mathbf{n}). \tag{1}$$

The Eulerian frame approach is also referred to as the mixed frame approach if the radiation intensity is measured in the Eulerian frame while the opacities σ (embedded in the interaction terms) are measured in the comoving frame [6,31]. The second difficulty arises because although the transport operators have a simple formulation in the mixed/Eulerian frame, the radiation quantities are still characterized by waves propagating at the speed of light c. This dynamical scale is significantly larger than the speed of sound a_∞ which characterizes the material quantities in the absence of radiation, such variation in propagation speeds defines the nature of multiscale waves associated with radiation hydrodynamics. It is important to note that the reference frame one chooses to take moments of the photon transport equation does not affect how one defines the material quantities. A mixed frame approach was adopted because the resulting equations most closely resemble a system of hyperbolic balance laws which is advantageous for constructing a Godunov-type method. The third difficulty occurs because in addition to defining balance laws for the radiation quantities, one must also add relativistic stiff source terms that are correct to $\mathcal{O}(u/c)$ to the right-hand-sides of the non-relativistic conservation laws for the material quantities (i.e., Euler equations). These source terms are stiff because of the variation in time and length scales associated with radiation hydrodynamical problems [38]. Having to contend with stiffness arising from some waves propagating at the speed of light as well as source terms having large magnitudes, it is obvious to see how such numerical difficulties make conventional techniques like operator splitting and the method of lines breakdown [27,28,56].

This paper presents a hybrid Godunov method that addresses the above numerical difficulties. The technique adopts a mixed frame approach, includes the appropriate frame-dependent terms to $\mathcal{O}(u/c)$, is implicit with respect to the fastest hyperbolic wave speeds, and semi-implicitly updates the stiff source terms. The paper proceeds in the following manner. After defining the full system of equations for radiation hydrodynamics, the paper discusses what dynamics characterize the various asymptotic limits. Then, the paper gives an overview of the hybrid Godunov method and explains certain numerical properties that the algorithm possesses. The next three sections present the main algorithmic ideas behind the hybrid Godunov method. Lastly, the paper describes numerical tests that demonstrate the technique to be stable, robust, and accurate across various parameter regimes.

2. Radiation hydrodynamics

As presented in Lowrie et al. [33] and Lowrie and Morel [31], the system of equations for radiation hydrodynamics can be non-dimensionalized with respect to the material flow scale so that one can compare hydrodynamical and radiation effects as well as identify terms that are $\mathcal{O}(u/c)$ [31,33]. This scaling gives two important parameters: $\mathbb{C}=c/a_\infty$ which measures