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Reduced-order modeling of transonic flows around an airfoil submitted to small deformations

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ABSTRACT

A reduced-order model (ROM) is developed for the prediction of unsteady transonic flows past an airfoil submitted to small deformations, at moderate Reynolds number. Considering a suitable state formulation as well as a consistent inner product, the Galerkin projection of the compressible flow Navier–Stokes equations, the high-fidelity (HF) model, onto a low-dimensional basis determined by Proper Orthogonal Decomposition (POD), leads to a polynomial quadratic ODE system relevant to the prediction of main flow features. A fictitious domain deformation technique is yielded by the Hadamard formulation of HF model and validated at HF level. This approach captures airfoil profile deformation by a modification of the boundary conditions whereas the spatial domain remains unchanged. A mixed POD gathering information from snapshot series associated with several airfoil profiles can be defined. The temporal coefficients in POD expansion are shape-dependent while spatial POD modes are not. In the ROM, airfoil deformation is introduced by a steady forcing term. ROM reliability towards airfoil deformation is demonstrated for the prediction of HF-resolved as well as unknown intermediate configurations.

1. Introduction

The increasing demand concerning the implementation of realistic flow simulations into iterative processes like real-time control, optimal shape design or parametrical studies, highlights the need for hierarchical modeling approaches. In this context, such approaches can provide an interesting balancing between physical accuracy and computational cost. Following this idea, high-fidelity (HF) models characterized by a high physical universality can be replaced by low-dimensional models that represent local approximations of HF models, allowing a strong reduction of the number of degrees of freedom.

The present work focuses on the development of a reduced-order model (ROM) for the prediction of unsteady transonic flows around an airfoil submitted to small deformations. The main contributions of the study are: (i) the elaboration of a ROM of the fully compressible flow non-linear Navier–Stokes equation system by Proper Orthogonal Decomposition (POD)-Galerkin approach, (ii) the introduction of small shape deformations in the ROM on the basis of the Hadamard formulation of HF model and (iii) the analysis of ROM reliability towards small shape deformations.

In the context of model reduction, the technique relying on the Galerkin projection of HF model onto a low-dimensional basis determined by POD, among other approaches, has been widely applied. The POD, also referred to as Principal Component Analysis [1] or Karhunen-Loève expansion [2], was initially utilized in fluid mechanics for coherent structure

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identification in turbulent flows [3]. It is often used to capture main flow features by a low number of basis functions or modes. As a consequence, POD-Galerkin approach is both *physics*- and *data-driven* since HF physical model is present through its Galerkin projection while POD modes are determined from flow snapshot series. The first aspect enforces the physical relevance of the approach while the second is responsible for both dimension reduction and local validity.

On the basis of Navier–Stokes equations and under incompressibility assumption, POD-Galerkin approach has been considered to derive ROMs of laminar and transitional flows predicted by direct numerical simulation [4–10], turbulent flows simulated by large eddy simulation [11] or statistical approaches [12], for example, as well as noisy laminar flows issued from stochastic simulations relying on polynomial chaos representations [13]. Compressible flows have been less investigated, especially in the non-linear fully compressible case. A framework based on POD-Galerkin approach has been reported in [14] for the linearized inviscid Euler equations. Adopting a linearization about a state determined by solving the non-linear governing equations (Euler or viscous-inviscid coupling), ROMs based on time-and frequency-domain POD have been elaborated for the prediction of transonic flows in turbomachinery and airfoil aeroelasticity context [15–18]. An aeroelastic POD ROM of a complete aircraft configuration in the transonic regime has been developed in the frequency domain on the basis of a linearized formulation of HF model [19]. A non-linear extension of frequency-domain POD-based ROM has been developed by means of automatic differentiation [20] and led to efficient prediction of inviscid transonic flows around an airfoil.

The present study concerns non-linear ROM of transonic flows in the time domain. In the fully compressible case, the coupling of the kinematic quantities with two thermodynamic variables induces two main difficulties concerning POD-Galerkin approach. The classical conservative formulation of the state vector does not lead to polynomial fluxes as in the incompressible case. This strongly complexifies the Galerkin projection and does not allow once for all computation of ROM coefficients. Moreover, in this context, the inner product usually considered for POD in the incompressible case is not dimensionally consistent. Under isentropic flow assumption, that is valid for moderate Mach numbers, a ROM of the compressible cavity flow has been put forward [21]. This assumption allows to express the governing equations as quadratic fluxes and an energetic inner product involving both flow and sound velocities can be defined. However, in the fully compressible case, the physical context of this work, the above mentioned difficulties have to be overcome. As reported in the present paper, the two key enablers are a modified state formulation along with constant viscosity assumption that lead to quadratic fluxes [22] and a consistent definition of the non-dimensional inner product as suggested in a preliminary work [23].

Beyond the elaboration of an efficient ROM for unsteady transonic flows, one of the main objectives of this study is to develop a ROM able to handle airfoil small deformations and reliable for the prediction of the effect of such deformations on predominant flow features. From a general point of view, a crucial issue in ROM development is the robustness of its predictive capacities in a certain neighborhood about reference configurations. Within such trust-regions, ROMs are expected to respond similarly to HF model. The integration of POD-Galerkin models into control procedures involving flow actuation [e.g. 24–27] or into parametrical studies concerning Reynolds number for example [4,6,8], has emphasized POD ROM sensitivity. These studies have highlighted inherent issues and limitations of this approach and suggested improvements, especially concerning POD basis validity and the introduction of flow actuation in the ROM. In the present work, these points are addressed in the case of small parametrical deformations of airfoil profile.

In the literature, only a few studies have dealt with ROM of flows around deformed bodies. The approaches based on an actual deformation of the computational grid have to face several issues concerning POD and especially the evaluation of spatial inner products between snapshots associated with different domains [28]. If the number of discretization points is unchanged during deformation, the 'index-based' POD approach [29] can be considered: the discretized POD modes are not associated with a specific spatial location but with space discretization point numbering. A drawback of this approach is the dependency of POD modes on the method considered to propagate the body deformation within the domain. As a consequence, for small deformations, considering a reference domain that is not modified by the deformation and introducing this deformation through a modification of the boundary conditions appears as a convenient alternative. In particular, the 'transpiration' method, that enforces the impermeability condition on the fictitious surface, has been used for both HF modeling [30,31] and ROM [32] on the basis of Euler equations. In the present physical context that concerns viscous flows in the transonic regime, a fictitious domain deformation technique is developed on the basis of the Hadamard formulation of HF model. As discussed in the following, this approach mimics efficiently airfoil deformation at HF level and yields a simple framework for introducing shape deformation in ROM.

The paper is organized as follows. HF model, related numerical method as well as the physical context of the study are briefly described in Section 2. A ROM of the transonic flow past NACA0012 airfoil is elaborated in Section 3 from this HF model via POD-Galerkin approach. In Section 4, a fictitious domain deformation technique is developed and validated at HF level. It is applied in Section 5 where ROM approach is extended to handle airfoil profile deformations. The main findings of the present work are summarized in Section 6.

2. High-fidelity model

The system of the compressible flow Navier–Stokes equations is considered as HF model for the prediction of the present transonic flows. The governing equations are briefly recalled (Section 2.1) as well as the numerical method (Section 2.2). The