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A two-dimensional unstructured cell-centered multi-material ALE scheme using VOF interface reconstruction

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ABSTRACT

We present a new cell-centered multi-material arbitrary Lagrangian–Eulerian (ALE) scheme to solve the compressible gas dynamics equations on two-dimensional unstructured grid. Our ALE method is of the explicit time-marching Lagrange plus remap type. Namely, it involves the following three phases: a Lagrangian phase wherein the flow is advanced using a cell-centered scheme; a rezone phase in which the nodes of the computational grid are moved to more optimal positions; a cell-centered remap phase which consists of interpolating conservatively the Lagrangian solution onto the rezoned grid. The multi-material modeling utilizes either concentration equations for miscible fluids or the Volume Of Fluid (VOF) capability with interface reconstruction for immiscible fluids. The main original feature of this ALE scheme lies in the introduction of a new mesh relaxation procedure which keeps the rezoned grid as close as possible to the Lagrangian one. In this formalism, the rezoned grid is defined as a convex combination between the Lagrangian grid and the grid resulting from condition number smoothing. This convex combination is constructed through the use of a scalar parameter which is a scalar function of the invariants of the Cauchy–Green tensor over the Lagrangian phase. Regarding the cell-centered remap phase, we employ two classical methods based on a partition of the rezoned cell in terms of its overlap with the Lagrangian cells. The first one is a simplified swept face-based method whereas the second one is a cell-intersection-based method. Our multi-material ALE methodology is assessed through several demanding two-dimensional tests. The corresponding numerical results provide a clear evidence of the robustness and the accuracy of this new scheme.

1. Introduction

Numerical schemes in compressible fluid dynamics make use of two classical kinematic descriptions: the Lagrangian description and the Eulerian description. Lagrangian algorithms are characterized by computational cells that move with fluid velocity. They allow an easy and natural tracking of free surfaces and interfaces between different materials. However, they suffer from a lack of robustness when they are facing large flow distortions. On the other hand, Eulerian algorithms are characterized by a fixed computational grid through which fluid moves. They can handle large distortions without any difficulties. However, the numerical diffusion inherent in advection terms discretization leads to an inaccurate interface definition and a loss in the resolution of flow details. The arbitrary Lagrangian–Eulerian (ALE) description has been initially introduced in the seminal paper [22] to solve in a certain extent the shortcomings of purely Lagrangian and purely Eulerian

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descriptions by combining the best features of both aforementioned approaches. The main feature of the ALE methodology is to move the computational grid with a prescribed velocity field to improve the accuracy and the robustness of the simulation. ALE methods have been used for several decades to face successfully the difficulties inherent in the simulation of multi-material fluid flows with large distortions [3,5,43,23,8,15,39,13]. Usually, ALE methods can be implemented in two manners. The first one, which is termed direct ALE, consists in an unsplit moving mesh discretization of the gas dynamics equations wherein the grid velocity is typically deduced from boundaries motion [44,36]. In this approach convective terms are solved directly. The second one, which is the subject of the present paper, is named indirect ALE. The main elements of an indirect ALE approach are an explicit Lagrangian phase in which the physical variables and grid are updated, a rezoning phase in which nodes of the Lagrangian grid are moved to improve the geometric quality of the grid and a remapping phase wherein the physical variables are conservatively interpolated from the Lagrangian grid onto the new rezoned one [40]. We point out that indirect ALE method encompasses both Lagrangian and Euler approaches. Indeed, when the rezoned mesh coincides with the initial mesh, indirect ALE algorithm corresponds to an Eulerian algorithm which is termed as Lagrange plus remap algorithm wherein advection terms are solved through the use of the remapping phase.

This paper aims at presenting a cell-centered indirect ALE algorithm to solve multi-material compressible flows on two-dimensional unstructured grids with fixed topology. Our Lagrangian phase solves the gas dynamics equations utilizing a moving mesh cell-centered discretization wherein the physical conservation laws are discretized in a compatible manner with the nodal velocity so that the geometric conservation law (GCL) is exactly satisfied [13]. Namely, the time rate of change of a Lagrangian volume is computed consistently with the node motion. This critical requirement is the cornerstone of any Lagrangian multidimensional scheme. Nowadays, cell-centered finite volume schemes [12,38,37] that fulfill this GCL requirement seem to be a promising alternative to the usual staggered finite difference discretization [11]. Moreover, these cell-centered schemes allow straightforward implementation of conservative remapping methods when they are used in the context of ALE. Here, we are using the high-order cell-centered Lagrangian scheme that has been described in [37]. Let us recall that the numerical fluxes are determined by means of a node-centered approximate Riemann solver. This discretization leads to a conservative and entropy consistent scheme whose high-order extension is derived through the use of generalized Riemann problem [7,37].

The thermodynamical modeling of multi-material flows in our ALE algorithm is considered through the use of two different approaches. In the first one, the multi-material flow is viewed as a multi-component mixture of miscible fluids wherein each fluid is characterized by its mass fraction, i.e. concentration. In this modeling, concentration stands for a passive scalar which allows to track the location of each material inside the flow. The mixture equation of state is obtained using a pressure–temperature equilibrium assumption. This modeling is quite simple to implement and to use. However, it can lead to inaccurate results as the numerical diffusion inherent in the concentration remapping may involve spurious numerical mixing. To correct this potential flaw, we have developed another approach which corresponds to the case of immiscible fluids. This second approach is based on the Volume Of Fluid (VOF) methodology which allows a Lagrangian tracking capability for material interfaces. Namely, contrary to concentration equations modeling, there is no mass flux between materials. The VOF approach introduces mixed or multi-material cells, which contain more than one material. Each material is characterized by its volume fraction, i.e. the ratio between the volume occupied by the material and the total volume of the mixed cell. We note that our implementation is restricted to two materials. The main issue related to mixed cell is define its evolution during the Lagrangian phase. To this end, we use a closure model that enables us to compute an effective thermodynamic state in terms of the thermodynamic states of each material and its related volume fraction. Here, we use the classical equal strain model [8], knowing that more sophisticated modeling are possible [4,24]. Knowing the volume fractions field, we perform a reconstruction of the interface in each cell by means of a piecewise linear representation which is obtained extending the well-known Youngs [51] algorithm to unstructured grids.

Essential for successful application of our ALE algorithm is the use of a good mesh rezoning strategy. This is not a simple task, since one has to balance between various requirements, some of which might seem to be contradictory. Generally, a proper rezoning strategy should maintain reasonable geometrical quality of the mesh while respecting the features of the underlying flow imprinted into the mesh deformation during the Lagrangian phase [28]. Since the objectives of mesh rezoning are close to the objectives of mesh generation, the rezoning strategies for ALE are closely related to the techniques developed and used by the mesh generation community. Here, we will restrict ourselves to rezoning by node repositioning, without changing the mesh connectivity. We point out that an original rezoning strategy has recently been developed where the connectivity of the mesh is allowed to change through the use of Voronoi tessellation. This new approach provides a Reconnection-based arbitrary Lagrangian–Eulerian (ReALE) strategy [34,35]. In the context of fixed topology, the geometric rezoning can easily be expressed as an optimization problem, where some mesh quality functional is minimized in order to find suitable mapping from the logical (computational) to the physical (real) space. Typically the functional contains information about smoothness of the mesh, its orthogonality, etc. A classical approach was originally proposed by Winslow [49,50] and is still considered to be the standard method. Here, we are making use of the condition number smoothness functional introduced in [30,29], which is closely related to Winslow smoothing, and is widely used on triangular and structured quadrilateral meshes. A generalized approach will be given, which can be applied to any unstructured meshes. An original relaxation procedure allows to define the rezoned grid as a convex combination between the Lagrangian grid and the regularized grid, i.e. the grid produced by the condition number smoothing. This convex combination is constructed through the use of an ω factor which is expressed in terms of the invariants of the right Cauchy–Green tensor [9] with respect to the Lagrangian displacement over a time