Journal of Computational Physics 229 (2010) 5843–5856

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Kinetic theory based lattice Boltzmann equation with viscous dissipation and pressure work for axisymmetric thermal flows

Lin Zheng, Zhaoli Guo *, Baochang Shi, Chuguang Zheng

National Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 18 October 2009
Received in revised form 7 February 2010
Accepted 14 April 2010
Available online 18 April 2010

Keywords:
Lattice Boltzmann equation
Axisymmetric thermal flow
Kinetic theory
Buoyancy-driven flow

ABSTRACT

A lattice Boltzmann equation (LBE) for axisymmetric thermal flows is proposed. The model is derived from the kinetic theory which exhibits several features that distinguish it from other previous LBE models. First, the present thermal LBE model is derived from the continuous Boltzmann equation, which has a solid foundation and clear physical significance; Second, the model can recover the energy equation with the viscous dissipation term and work of pressure which are usually ignored by traditional methods and the existing thermal LBE models; Finally, unlike the existing thermal LBE models, no velocity and temperature gradients appear in the force terms which are easy to realize in the present model. The model is validated by thermal flow in a pipe, thermal buoyancy-driven flow, and swirling flow in vertical cylinder by rotating the top and bottom walls. It is found that the numerical results agreed excellently with analytical solution or other numerical results.
© 2010 Elsevier Inc. All rights reserved.

1. Introduction

The axisymmetric thermal flows in the axisymmetric system is of great interest in fluid mechanics [1–3]. In the last two decades, LBE has been rapidly developed as an effective and promising numerical algorithm for computational fluid dynamics [4–6], which has also been applied to axisymmetric flows [7–19]. The straightforward way for LBE to simulate such flows is using a 3D LBE model with suitable curved boundary treatments [22–24]. Nevertheless, such approach implies the expensive computational costs for this 3D simulation which does not consider any symmetrical properties of the axisymmetric flows. Considered the properties of the axisymmetric flows, such flows can be reduced to a quasi-two-dimensional problems in the meridian plane. Although the LBE method has achieved great success in simulating axisymmetric athermal/isothermal fluid flows, it still has many challenges for constructing an effective and applicable axisymmetric LBE model.

In the literature, there are three categories of axisymmetric LBE models proposed for axisymmetric athermal flows [10–16], i.e., the coordinate transformation method (CTM), the vorticity–stream method (VSM) and the double-distribution-function (DDF) method. The main idea of CTM is that it transforms the axisymmetric Navier–Stokes equations (NSE) to the specific pseudo-Cartesian forms with some additional terms in these quasi-two-dimensional NSE. The CTM was introduced by Halliday et al. [10], who first proposed an axisymmetric D2Q9 model by adding some source terms into LBE so that it could recover the axisymmetric NSE at the macroscopic level. However, Lee et al. [11] found that some terms are missing in this model which would lead to large errors for simulating the constricted or expanded pipe flows. Later, Reis and Phillips [12,13] and Zhou [14] developed similar models based on the axisymmetric NSE. On the other hand, the VSM is another version of transformation method, it uses the relations between the vorticity, the stream function and the velocity, then the 3D axisymmetric NSE can be transformed to the vorticity–stream-function equations. Based on the vorticity–stream-function

* Corresponding author.
E-mail address: zlguo@hust.edu.cn (Z. Guo).

0021-9991/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcp.2010.04.026

equations, Chen et al. [15] constructed a LBE model, but the model must solve a Poisson equation at each time step. It should be mentioned that the effect of azimuthal velocity is neglected by most of the above mentioned LBE models. Recently, considered the effect of the azimuthal velocity, the DDF method is first proposed by Guo et al. [16] for simulating athermal axisymmetric flow from the Boltzmann equation, one is for solving the axial and radial velocity components, and the other is for solving azimuthal velocity. This DDF method has a solid theory foundation and clear physical significance, and the main difference between Guo et al.'s model and the other existing models is that Guo et al.'s model is designed in a bottom-up fashion but the other existing models are designed in a top-down fashion.

However, the aforementioned LBE models are restricted to athermal axisymmetric flows, while very few axisymmetric LBE models proposed for axisymmetric thermal problems. To the authors' knowledge, only four works [17–20] applied the LBE to the axisymmetric thermal flows. These models can be classified into two categories, i.e., the hybrid approach and the double-distribution-function (DDF) method. The hybrid method is directly applied athermal axisymmetric LBE model to simulate the flow field, while the energy equation is solved by different numerical methods rather than solving LBE [17,18]. The DDF method utilizes two different density distribution functions similar to the athermal DDF approach, one is for solving the vorticity–stream-function equations or the NSE, and the other is for solving the energy equation [19,20]. The main difference between the DDF method and the hybrid method is that the energy equation is solved by LBE.

The application of hybrid approach and DDF method for axisymmetric thermal flows encountered many challenges. The fundamental problem with most of the hybrid LBE models [17,18] is that they directly used the athermal axisymmetric CTM LBE models to solve the flow field, which have the complicated force terms. Furthermore, due to the complicated force terms, the numerical instability is another critical problem for these hybrid methods, although the stability is improved by the modified model [18]. On the other hand, although the DDF approach proposed by Chen et al. [19] has greatly improved the numerical stability, and the complex force terms have been simplified, the model has to solve the Poisson equation at each time step as the limitation of their athermal model. Moreover, in Ref. [20], Zheng et al. pointed out this model seems to mismatch the energy equation at the macroscopic level, and proposed another version of DDF LBE model which can overcome this problem, and the force terms in the model have no velocity and temperature gradients. However, it should be pointed out that the existing axisymmetric thermal LBE models usually ignore the effect of viscous dissipation term and work of pressure in the energy equation. Therefore, it is desirable to construct a more general axisymmetric LBE model for axisymmetric thermal flows.

In sight of the limitations in the previous works, in this paper, we aim to propose an axisymmetric thermal LBE model from the continuous Boltzmann equation, which could recover the energy equation with the viscous dissipation term and work of pressure. The rest of the paper is organized as follows. In Section 2, kinetic theory of axisymmetric Boltzmann equation is introduced. In Section 3, the axisymmetric thermal LBE model derived from the continuous Boltzmann equation, and some numerical tests of the LBE model are conducted in Section 4, and finally a brief conclusion is presented in Section 5.

2. Kinetic theory of axisymmetric Boltzmann equation

The fully axisymmetric Boltzmann equation including an external force with the Bhatnagar–Gross–Krook (BGK) collision operator for symmetric flows is given as

$$\frac{\partial f}{\partial t}+\xi_x\frac{\partial f}{\partial x}+\xi_r\frac{\partial f}{\partial r}+\frac{\xi_\theta^2}{r}\frac{\partial f}{\partial \xi_r}-\frac{\xi_r\xi_\theta}{r}\frac{\partial f}{\partial \xi_\theta}+\boldsymbol{a}_0\cdot\frac{\partial f}{\partial_{\xi_0}}=-\frac{1}{\tau_f}[f-f^{(eq)}], \tag{1}$$

where $f(\boldsymbol{x},\xi_0,t)\equiv f(x,r,\xi_x,\xi_r,\xi_\theta,t)$ is the density distribution function of fluid molecules moving with velocity $\xi_0=(\xi_x,\xi_r,\xi_\theta)$ at position $\boldsymbol{x}=(x,r)$ and time t in the cylindrical coordinates, $\boldsymbol{a}_0=(a_x,a_r,a_\theta)$ is the external force, τ_f is relaxation time and $f^{(eq)}$ is local Maxwellian equilibrium distribution defined by

$$f^{(eq)}=\frac{\rho}{(2\pi RT)^{3/2}}\exp\left[-\frac{|\xi_0-\boldsymbol{u}_0|^2}{2RT}\right], \tag{2}$$

where R is gas constant, ρ, $\boldsymbol{u}_0=(u_x,u_r,u_\theta)$ with u_x, u_r and u_θ being axial, radial and azimuthal velocity components, and T are respectively the fluid density, velocity and temperature defined by,

$$\rho=\int f\,d\xi_0,\quad \rho\boldsymbol{u}_0=\int \xi_0 f\,d\xi_0,\quad 3\rho RT=\int|\xi_0-\boldsymbol{u}_0|^2 f\,d\xi_0, \tag{3}$$

As the similar procedure as Ref. [16], we introduce the following reduced distribution functions

$$\tilde{f}(\boldsymbol{x},\xi)=\int f\,d\xi_\theta,\quad \bar{f}(\boldsymbol{x},\xi)=\int \xi_\theta f\,d\xi_\theta,\quad \hat{f}(\boldsymbol{x},\xi)=\int\frac{|\xi_0|^2}{2}f\,d\xi_\theta, \tag{4}$$

and then Eq. (1) can be simplified to the following three equations

Download English Version:

https://daneshyari.com/en/article/521170

Download Persian Version:

https://daneshyari.com/article/521170

Daneshyari.com