Journal of Computational Physics 245 (2013) 160–204

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Fully conservative leak-proof treatment of thin solid structures immersed in compressible fluids

Jón Tómas Grétarsson *, Ron Fedkiw

Stanford University, 353 Serra Mall Room 207, Stanford, CA 94305, United States

ARTICLE INFO

Article history:
Received 8 March 2012
Received in revised form 19 December 2012
Accepted 1 February 2013
Available online 28 February 2013

Keywords:
Fluid–structure interactions
Compressible flow
Leakproof
Thin structures

ABSTRACT

We propose a novel high resolution conservative advection scheme that is suitable for thin, embedded moving solid structures. The scheme works by coupling together a high order flux-based method with a conservative semi-Lagrangian solver that is similar in spirit to that of Lentine et al. [26], but modified to treat the cut cells and partial volumes that arise near a thin solid structure. The conservative semi-Lagrangian scheme is unconditionally stable, and so unlike previous methods no cell merging is required to compensate for the small cell volumes that arise. Furthermore, as the semi-Lagrangian scheme works via tracing characteristic curves, no special treatment is required either to enforce non-penetration through thin, moving solid structures, or to populate swept or uncovered degrees of freedom. For the flux-based solver, we use finite-difference ENO with Lax–Friedrich's diffusion (although any flux-based scheme works), and in doing so we found that a modification to the diffusion calculation leads to improved stability in its third order accurate variant. We integrate this novel hybrid advection scheme into a semi-implicit compressible flow solver, and modify the implicit pressure solver to work with cells of variable size. In addition, we propose an improvement to the semi-implicit compressible flow solver via a new method for computing a post-advected pressure. Finally, this hybrid conservative advection scheme is integrated into a semi-implicit fluid–structure solver, and a number of one-dimensional and two-dimensional examples are considered—in particular, showing that we can handle thin solid structures moving through the grid in a fully conservative manner, preventing fluid from leaking from one side of the structure to the other and without the need for cell merging or other special treatment of cut cells and partial volumes.

1. Introduction

The Direct Numerical Simulation (DNS) of fluid–structure interactions has recently received significant attention. Many of these works concern themselves with fluid flow in the incompressible flow regime, see for example [8,22] and the references within, but researchers are increasingly giving attention to the two-way coupled interactions that arise in compressible flows, see for example [4,17,9]. If one desires to use a state-of-the-art Eulerian method on the fluid flow, and a state-of-the-art Lagrangian method for the structure solver, then this requires a numerical method for coupling these two solvers together. Fluid-based forces need to be transferred to the solid structure, and position and velocity-based boundary conditions must be applied to the fluid based on the current location and movement of the solid structure. One of the primary research areas in solid–fluid coupling concerns the stability of the numerical methods for coupling and is essentially focused on the feedback loop where pressure is applied to the solid, the solid structure reacts and deforms, and subsequently

* Corresponding author.
E-mail addresses: jontg@cs.stanford.edu (J.T. Grétarsson), fedkiw@cs.stanford.edu (R. Fedkiw).

0021-9991/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jcp.2013.02.017

imposes position and velocity-based boundary conditions on the fluid. While the most straightforward approach is simply to treat the coupling in an explicit way, called a partitioned method [49,41,10], researchers have focused quite a bit of attention on so-called monolithic methods that employ higher degrees of implicit coupling [42,14], in order to stabilize parts or all of this feedback loop. Another important issue regards the modifications that the Eulerian method requires to treat cells cut by the solid structure as well as those that are covered or uncovered as the structure sweeps across the Eulerian grid—especially in regards to stability and conservation. A common approach for treating these issues on the Eulerian grid is to fill the cells that are covered or partially covered by the solid structure with ghost values of some type, and then proceed in the standard way ignoring the solid all-together. This alleviates stability restrictions for cut cells, automatically creates new fluid in uncovered cells, and has been theme of the approach for the ghost fluid method [11] and the immersed boundary method (see [40] and the references therein, including [38,39]). The fluid placed in these ghost cells must include the added mass effect of the solid, i.e. if the solid is heavier or lighter than the surrounding fluid, the ghost cells must properly represent that mass difference. The added mass can be accounted for in thin solid structures as well (see for example [50]), simply by adding that mass to the fluid cells that contain the solid structure. Whereas cut cell methods overcome stability restrictions for the cut cells, they do not maintain either conservation nor the ability for the fluid on one side of the structure to remain on that side, i.e. the fluid can leak across to the other side of the structure. In order to address these concerns, authors have focused on cut cell methods, see for example [16,17] and the references therein. The main issue with these methods is in the treatment of small cell volumes, which can impose additional time step restrictions on the flow solver if special techniques such as cell merging near the structure interface are not used. Furthermore these methods can become extremely complex if the solid structure is sweeping across the grid. In fact, most approaches to treating covering and uncovering of cells are non-conservative, and even then there can be issues—for example, [45] pointed out that this can cause pressure oscillations in incompressible flows. Generally speaking uncovered cells need to be replaced with a valid value, and one can do this with any number of methods that range from simply interpolating from nearby neighbors to using upwind information to populate these cells, see for example [28,27,47]. Our semi-Lagrangian approach also uses upwind information to fill uncovered cells, but with the aide of [26] more readily lends itself to a fully conservative approach than does a flux-based method.

We propose a novel treatment for cut cells and partial cell volumes near the structure interface. Unlike previous methods, this approach does not rely on cell merging to alleviate the time step restriction; instead we employ a conservative semi-Lagrangian scheme, similar in spirit to [26]. We make two major modifications to this method. First, the method is modified to support non-uniform grids, noting that care must be taken when a characteristic emanating from a large grid cell lands in the midsts of many small grid cells, and vice versa. Second, since the semi-Lagrangian method is low order accurate, we hybridize it with a high order accurate flux-based ENO method [46] (although any flux-based scheme works) so that high resolution can be obtained throughout the flow with the semi-Lagrangian method only being applied near the thin solid interface. As the semi-Lagrangian solver works by tracing along characteristic curves, we can use continuous collision-detection [7,15] to guarantee that fluid does not penetrate into a volumetric solid or cross over from one side to the other on a thin solid. This works even when the solid is moving and is under-resolved by the grid. Notably, the resulting method requires no special treatment for swept or uncovered cells.

Using the semi-Lagrangian method to handle cells near the structure interface is similar in spirit to both volume of fluid (VOF) [20] and arbitrary Lagrange–Eulerian (ALE) [19] methods, which both explicitly move information along characteristics in a Lagrangian manner and both explicitly conserve the material. Although some versions of the volume of fluid scheme intersect flux-swept volumes with the volume fraction, others actually mesh up the volume fraction and move it through the grid in a Lagrangian fashion. If one treats each vertex of the meshed-up VOF polygon as a Lagrangian particle, continuous collision-detection can be applied to it in the same fashion as we do for our semi-Lagrangian rays. In this manner one can achieve conservation, stability and also prevent material from interpenetrating volumetric solids or crossing over thin solids. Afterwards, this advected polygon of volume needs to be deposited and stored on the grid so that it can be remeshed into the VOF representation at the next time step. The issue here comes in the representation; that is, if a cell is cut by a thin structure one needs to represent that volume fraction on the grid in a way that does not cross over the structure. The semi-Lagrangian method stores information at grid points (cell centers in our implementation) and therefore overcomes this, but a volume of fluid method would need to reconstruct the geometry in such a way that cuts the cells across the interface designated by the solid boundary. Similarly ALE methods push along the vertices of their mesh in a manner similar to both the VOF and semi-Lagrangian methods, and thus those vertices can be collided with the structure. Again, one of the more complex aspects of this is in keeping the structure for the ALE mesh commensurate with the solid structure interface. Moreover, another issue with the ALE method is that pushing nodes around in a Lagrangian fashion and colliding them with the structure interface can result in inversion, and unless one wants to untangle the ALE mesh [48] and attempt to fit it to the solid structure, a remapping method needs to be employed where the material is dropped back down onto some Eulerian mesh and then remeshed in a way that fits the structure. In general we believe that both VOF and ALE methods could be applied in a manner similar to what we propose for our method, as long as one could work out the details for hybridization with the flux-based scheme and for redepositing the material near the solid interface onto an Eulerian grid. However, we feel that the conservative semi-Lagrangian approach of [26] is a very simple and straight-forward way to do this. We refer the interested reader to the following relevant VOF [18,37,2,3,30] and ALE papers [23,34,33,35,36,5].

In order to capture the fluid–structure interactions we employ the flux-split compressible coupling methodology of [14], where the fluid flux terms are split into advective terms and pressure terms. The linearly degenerate advective terms are solved independently of the structure after which an implicit, monolithic coupled system is solved for the fluid pressures,