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Numerical upscaling for the eddy-current model with stochastic magnetic materials

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Abstract

This paper deals with the upscaling of the time-harmonic Maxwell equations for heterogeneous media. We analyze the eddy-current approximation of Maxwell's equations to describe the electric field for heterogeneous, isotropic magnetic materials. The magnetic permeability of the materials is assumed to have random heterogeneities described by a Gaussian random field. We apply the so-called Coarse Graining method to develop a numerical upscaling of the eddy-current model. The upscaling uses filtering and averaging procedures in Fourier space which results in a formulation of the eddy-current model on coarser resolution scales where the influence of sub-scale fluctuations is modeled by effective scale- and space-dependent reluctivity tensors. The effective reluctivity tensors can be obtained by solving local partial differential equations which contain a Laplacian as well as a curl–curl operator. We present a computational method how the equation of the combined operators can be discretized and solved numerically using an extended variational formulation compared to standard discretizations. We compare the results of the numerical upscaling of the eddy-current model with theoretical results of Eberhard [J.P. Eberhard, Upscaling for the time-harmonic Maxwell equations with heterogeneous magnetic materials, Physical Review E 72 (3), (2005)] and obtain a very good agreement.

Keywords: Eddy-current model; Numerical upscaling; Magnetic permeability; Stochastic modeling; Numerical discretization

1. Introduction

Many realistic electromagnetic systems show electric field effects which are strongly influenced by microscopic magnetic parameters of the materials. The phenomena of eddy-currents in heterogeneous magnetic materials is one of the examples where the system behavior depends on the microscopic magnetic permeability distribution. The electromagnetic interactions may take place on very small scales, often of atomistic magnitude. To reduce the computational complexity of the electromagnetic problem we are interested in an

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upscaling of the eddy-current model onto coarser length scales so that the impact of the sub-scale information need not to be modeled in detail. For the eddy-current model the resulting macroscopic field and current distribution strongly depend on the given magnetic permeability data of the material. However, the detailed magnetic permeability distribution is in most cases not known explicitly. As a solution the magnetic permeability in the eddy-current approximation can be described by the stochastic modeling. The stochastic modeling yields a general approach to handle problems involving heterogeneous materials. It describes an ensemble of realizations of the heterogeneous material by a random field. The material parameter can then be given by a realization of the random field where the values of the realization are explicitly known.

Upscaling for Maxwell's equations has been used for a long time. The use of the mathematical homogenization theory is one of the first approaches which results in averaged or homogenized equations on a coarser scale [20]. For materials with micro-periodic structure a homogenization technique was first introduced in [6,15], where the homogenized material properties have been found by solving local problems by suitable averaging. Heterogeneous composite materials have widely been analyzed, see e.g. [23,24]. A scaling theory for Maxwell's equation concerning composite material was developed in Ref. [31] for the first time. In general, all these homogenization methods have been well analyzed in the literature. For practical applications, however, they may be of limited use due to the more theoretical approach, the requirements of periodic structures, as well as the upscaling to a fixed scale only. For many practical applications, these restrictions may be too far-ranging. Moreover, natural phenomena often need to be modelled by a more realistic way assuming a stochastical distribution for the heterogeneities of the material, where the material characteristics are supposed to be upscaled to various length scales. In this case other upscaling procedures have to be applied. In Ref. [10], a wavelet-base upscaling method was analyzed suitable for non-periodic material. For flow problems an upscaling procedure called Coarse Graining method is well known which allows to perform analytical investigations of heterogeneous media. In the study [11], Eberhard applied the Coarse Graining method for the first time to develop an upscaling for the time-harmonic Maxwell equations with stochastic magnetic materials.

The Coarse Graining method was originally derived in the scope of large eddy simulations and for flow in heterogeneous media, see [25,3,2]. Its basic idea is a splitting and averaging of high-frequency modes in Fourier space. The influence of sub-scale fluctuations is given by effective material parameter tensors. The unique advantage of the Coarse Graining method is that nearly any material can be treated without any restrictions such as periodicity. Also, the scale of the upscaling can be chosen arbitrarily in contrast to the homogenization theory.

The theory of the Coarse Graining method for Maxwell's equations has been analyzed in Ref. [11] where theoretical results for the reluctivity tensors are obtained via a second-order perturbation theory. The numerical computation of these tensors is still lacking, which will be addressed in this paper. We therefore analyze the eddy-current approximation with heterogeneous, isotropic magnetic materials, and we formulate the numerical upscaling based on the Coarse Graining method as shown in Ref. [11]. The upscaling results in effective scale- and space-dependent reluctivity tensors which can be computed by solving local partial differential equations – the so-called sub-problems. Due to the Laplacian and the curl–curl operator these equations are hard to analyze computationally. We present a computational method to discretize the combined operators and to solve them numerically using an extended variational formulation.

The paper is organized as follows. The next section summarizes the physical and mathematical methods to formulate the numerical upscaling. In particular, we introduce the modeling of statistically distributed data with stochastic fields. In Section 3, the theory of the Coarse Graining method for time-harmonic Maxwell equations is introduced as developed by Eberhard [11]. We continue his study and present a numerical approach for the upscaling. To calculate the upscaled material parameters we need to solve local partial differential equations. Section 4 shows the detailed variational formulation and discretization to obtain a computational scheme. In Section 5, we present numerical results of the upscaling and compare them with the theoretical results given by the perturbation theory. We finish with a conclusion and outlook.

2. Definitions and mathematical statement

We briefly describe the mathematical background and the formulation underlying the upscaling for the time-harmonic Maxwell's equation in the eddy-current approximation. In particular, we introduce the