Journal of Computational Physics 229 (2010) 4225–4246

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Handling solid–fluid interfaces for viscous flows: Explicit jump approximation vs. ghost cell approaches

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ARTICLE INFO

Article history:
Received 18 February 2009
Received in revised form 7 February 2010
Accepted 8 February 2010
Available online 14 February 2010

Keywords:
Explicit jump approximation
Ghost cell approach
Viscous flow
Diffusion equation
Heat equation
Jump correction
The immersed interface method
Irregular solid boundary
The polygonal area mapping method
HyPAM

ABSTRACT

The ghost cell approaches (GCA) for handling stationary solid boundaries, regular or irregular, are first investigated theoretically and numerically for the diffusion equation with Dirichlet boundary conditions. The main conclusion of this part of investigation is that the approximation for the diffusion term has to be second-order accurate everywhere in order for the numerical solution to be rigorously second-order accurate. Violating this principle, the linear and quadratic GCAs have the following shortcomings: (1) restrictive constraints on grid size when the viscosity is small; (2) susceptibleness to instability of a time-explicit formulation for strongly transient flows; (3) convergence deterioration to zeroth- or first-order for solutions with high-frequency modes. Therefore, the widely-used linear extrapolation for enforcing no-slip boundary conditions should be avoided, even for regular solid boundaries. As a remedy, a simple method based on explicit jump approximation (EJA) is proposed. EJA hinges on the idea that a velocity-derivative jump at the boundary reduces to the value of the velocity-derivative at the fluid side because the velocity of the stationary boundary is zero. Although the time-marching linear system of EJA is not symmetric, it is strictly diagonal dominant with positive diagonal entries. Numerical results show that, over a large range of viscosity and grid sizes, EJA performs much better than GCAs in terms of stability and accuracy. Furthermore, the second-order convergence of EJA does not depend on viscosity and the spectrum of the solution, as those of GCAs do. This paper is written with enough details so that one can reproduce the numerical results.

1. Introduction

For any irregular domain with smooth boundaries, a smooth function can be extended across a boundary with a bound on the relative increase in the error norms. This is the essential idea behind the ghost cell approach (GCA). With the ghost cell values updated by extrapolating from inside the fluid phase, the boundary conditions at the solid–fluid interface are implicitly fulfilled. This approach dates back to Mayo [13] in 1980s and has been widely adopted by researchers; one group of examples [18,1] concerns the immersed boundary (IB) method with the finite difference formulation.

There exist numerous ways of obtaining the ghost cell values, most of them are variants of two formulas: the linear extrapolation via image points, hereafter referred to as GC1, and the quadratic extrapolation via polynomial fitting, hereafter referred to as GC2. Both GC1 and GC2 have been widely used in treating irregular boundaries, see [20,18] for two examples and [16] for a wide perspective.

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0021-9991/$ - see front matter
doi:10.1016/j.jcp.2010.02.007

This paper considers the nonhomogeneous diffusion equation,

$$\frac{\partial u}{\partial t} = \nu \nabla^2 u + f(\mathbf{x}, t), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^{\mathbf{D}}$ is the location vector, t the temporal coordinate, $u = u(\mathbf{x}, t)$ a continuous scalar function with its value being a constant zero within the solid phase, ν the dynamic viscosity, and the forcing term $f(\mathbf{x}, t)$ is known *a priori*. In this and the next sections, we will focus on the one-dimensional version of (1):

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + f(y, t), \tag{2}$$

where y denotes the vertical coordinate. The solid–fluid interface is located at $y_B = bh$, with h being the uniform mesh spacing, as shown in Fig. 1. The regular boundary case in Fig. 1(a) can be considered as a special case of the irregular boundary case in Fig. 1(b) with $b = 0$. Without loss of generality, it is assumed that $b \in \left[-\frac{1}{2}, \frac{1}{2}\right)$ and the discretization of u is cell-centered. The value of u in the jth cell is represented by u_j, located at $y_j = \left(j + \frac{1}{2}\right)h$.

Referring to Fig. 1(b), the image point of y_{-1} with respect to the interface is located at $y_{\text{image}} = \left(2b + \frac{1}{2}\right)h$, where a linear interpolation yields $u_{\text{image}} = 2bu_1 + (1 - 2b)u_0$. GC1 sets the ghost cell value by

$$u_{-1}^{\text{GC1}} = 2u_B - u_{\text{image}}. \tag{3}$$

When $b = 0$ (i.e. regular boundary) and $u_B = 0$, (3) reduces to the well-known no-slip condition $u_{-1} = -u_0$ for regular boundaries, as shown in Fig. 1(a). Since (3) is linear, using a higher-order interpolation for u_{image} does not improve the overall accuracy unless more image points are introduced.

In GC2, the ghost cell value is evaluated by fitting a quadratic polynomial near the interface:

$$u_{-1}^{\text{GC2}} = \tilde{u}\left(-\frac{h}{2}\right), \tag{4}$$
$$\tilde{u}(y) = \frac{\left(y - \frac{3}{2}h\right)\left(y - \frac{5}{2}h\right)}{\left(b - \frac{3}{2}\right)\left(b - \frac{5}{2}\right)h^2} u_B + \frac{\left(y - \frac{5}{2}h\right)(y - bh)}{h^2\left(b - \frac{3}{2}\right)} u_1 - \frac{\left(y - \frac{3}{2}h\right)(y - bh)}{h^2\left(b - \frac{5}{2}\right)} u_2,$$

where the irregular cell value u_0 is excluded to prevent instabilities from $b \approx \frac{1}{2}$.

Taylor expansions of (3) and (4) at y_0 yield

$$\frac{u_{-1}^{\text{GC}k} + u_1 - 2u_0}{h^2} - \left.\frac{\partial^2 u}{\partial y^2}\right|_0 = T^{\text{GC}k} \left.\frac{\partial^{k+1} u}{\partial y^{k+1}}\right|_0 + O(h^k), \tag{5}$$

where

$$T^{\text{GC1}} = \frac{-1 - 8b + 4b^2}{4} \tag{6a}$$
$$T^{\text{GC2}} = \frac{1 + 2b}{2} h \tag{6b}$$

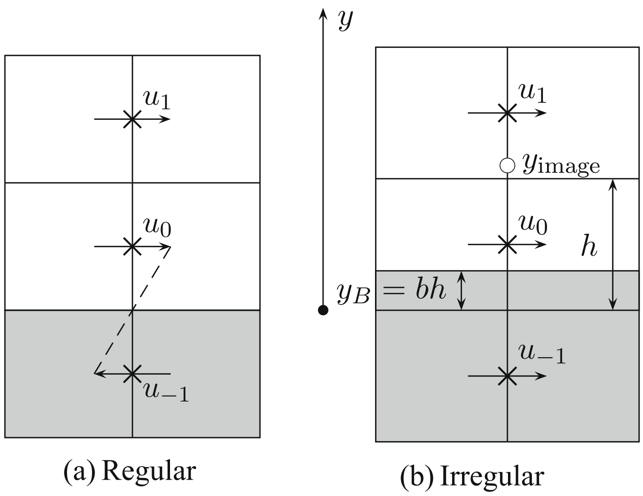

Fig. 1. Cells in the vicinity of the solid–fluid interface. Light gray area represents the solid phase and white area is occupied by the fluid phase. u_{-1} is a boundary condition to be specified. '$\circ$' represents the image point of y_{-1}.