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Modified interpolation kernels for treating diffusion and remeshing in vortex methods

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Abstract

A scheme treating diffusion and remeshing, simultaneously, in Lagrangian vortex methods is proposed. The vorticity redistribution method is adopted to derive appropriate interpolation kernels similar to those used for remeshing in inviscid methods. These new interpolation kernels incorporate diffusion as well as remeshing. During implementation, viscous splitting is employed. The flow field is updated in two fractional steps, where the vortex elements are first convected according to the local velocity, and then their vorticity is diffused and redistributed over a predefined mesh using the extended interpolation kernels. The error characteristics and stability properties of the interpolation kernels are investigated using Fourier analysis. Numerical examples are provided to demonstrate that the scheme can be successfully applied in complex problems, including cases of nonlinear diffusion.

Keywords: Numerical simulation; Computational particle methods; Vortex methods; Redistribution; Diffusion

1. Introduction

Lagrangian vortex methods [6,25] are tools for computing complex fluid flows. Several of the computational advantages of these methods are:

(1) While Eulerian methods introduce extra dispersion or dissipation, even in flows with zero velocity gradient, such errors are minimized during advection in Lagrangian vortex methods.
(2) The condition of numerical stability is not restricted by the CFL condition.
(3) The support of particle distribution remains a small fraction of the total volume of the flow field, determined by where vorticity is confined. The method is endowed with natural 'grid adaptivity', and hence the computational elements are utilized more efficiently.

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(4) The method provides a natural way to represent small vortical structures that arise at high Reynolds numbers.

While Lagrangian vortex methods were originally formulated for inviscid flows, successful approaches for viscous flows have been proposed [5,8,9,11,29,31]. In some methods, such as random walk [5] and diffusion velocity methods [11], particles are transported while their strength remains fixed. In other methods, the strength assigned to each particle is allowed to change without displacing the particles. In many cases, more particles are introduced to capture the expanding region where vorticity is confined.

One popular algorithm is the PSE (particle strength exchange) scheme [9], in which the diffusion equation is converted into integro-differential equations, which are discretized in space by approximating the integral using a quadrature rule. The semi-discrete equations are again discretized in time in various different ways—implicitly or explicitly—up to whatever order of accuracy is desired. This method has been successfully applied to several complex flows [19,27,39,41], and has been extended to the case of anisotropic diffusion [10], and to the case with spatially variable radius of the cutoff function [7].

The use of a quadrature rule in PSE requires relatively uniform particle distribution, and this naturally necessitates frequent remeshing. Remeshing is also implemented in other methods, even in inviscid simulations to satisfy other conditions. For instance, it has been observed that long-time accuracy of convection computation deteriorates severely due to the distortion of the particle distribution [6,14]. Several local regridding schemes have been devised to solve this problem, by inserting new particles where inter-particle distance becomes too large [16,17,41]. These schemes are limited to geometrically simple flows, and tend to grow the number of particles rapidly, unless careful clustering and merging is also implemented. For these reasons, global remeshing is now considered necessary in most Lagrangian particle methods, and the design and verification of various remeshing schemes have become an active research area [1,3].

In this article, we design a scheme that treats diffusion and remeshing simultaneously and without additional ambiguity or computational overhead. The scheme, 'redistribution onto a grid', will be formulated as an extension of the vorticity redistribution method [33], and cast in the form of interpolation kernels, which resemble those used in inviscid remeshing [6,18].

The paper is organized as follows. In Section 2, the vorticity redistribution method is introduced. Next, we develop the modified interpolation kernels in Section 3. The error characteristics and the stability properties of these kernels are investigated in Section 4. We finally provide numerical examples in Section 5.

2. The redistribution method

The vorticity redistribution method, or simply the redistribution method, developed in [33] is a deterministic approach to solve the constant-diffusivity diffusion equation. In this method, the fundamental solution of the diffusion equation for each particle vorticity is approximated by a new set of particles within a ball of a finite radius, whose locations and strengths are determined by satisfying a number of 'predictive moment matching conditions'. The latter enforce the requirement that the vorticity assigned to the new particles have approximately the same moments, up to a certain order, as the moments of the fundamental solution generated by the source particle. The new particle vorticity is obtained by redistributing the source particle strength onto the target particles, i.e., by transferring fractions of the source particle strength to the target particles nearby. The spatial resolution of the method is naturally defined by the redistribution radius, that is, the radius of the ball in which the target particles for each source particle lie.

How to obtain a redistribution formula that determines the correct redistribution fractions that satisfy the predictive moment matching conditions depends on the specific problem of interest. When the fundamental solution of the diffusion equation is known explicitly, the moments of the fundamental solution can be exactly determined, and the corresponding redistribution formula can be easily constructed [33]. However, for spatially varying or anisotropic diffusion, the explicit form of the fundamental solution is often not available. To address this difficulty, a more general method to design redistribution formulae satisfying the moment matching conditions was proposed [13,32], in which the evolution equations for the moments of the fundamental solution of each source particle were discretized by explicit integration schemes,