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Connectivity-free front tracking method for multiphase flows with free surfaces

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ABSTRACT

In this study, a connectivity-free front tracking method is developed to simulate multiphase flows with free surfaces. This method is based on the point-set method which does not require any connectivities between interfacial points to represent the interface. The main advantage of the connectivity-free approach is the easiness in re-constructing the interface when large topology change occurs. It requires an indicator field to be constructed first based on the existing interface and the surface curvature and normal are then computed using the indicator field. Here, we adopt the reproducing kernel particle method (RKPM) interpolation function that provides the ability to deal with free-surface flows and the flexibility of using non-uniform meshes when local fine resolution is needed. A points regeneration scheme is developed to construct smooth interfaces and to automatically handle topology changes. The mass conservation is verified by performing a single vortex advection test. Several 2-D and 3-D numerical tests including an oscillating droplet, dam-breaking, two droplet impacting and multi-bubble merging are presented to show the accuracy and the robustness of the method.

1. Introduction

Multiphase flows are involved in a wide range of applications in engineering fields such as liquid sprays, fuel tank sloshing, molding, bubble column reactors, steam generators and turbines, hydraulic design of dams, design of ship hulls, etc [1–3]. However, despite great efforts, developing a numerical technique that can accurately simulate multiphase flows involving moving interfaces still poses challenges. Issues such as the fluid density, viscosity and pressure jumps across the interface have been addressed using continuous or discontinuous numerical methods [4,5]. Studies have also been performed on modeling a smooth interface on discretized grids to treat interface topology changes such as coalescence and breaking-up [6].

There are two commonly used computational approaches to solve two-phase incompressible flows: the front capturing method and the front tracking method. The front capturing method usually includes the volume of fluid (VOF) method [7,8] and the level-set method [9,10]. The basic idea is to model the interface as the isosurface of a scalar function such as the indicator (color) function in VOF or signed distance function in level-set. Special advection scheme is adopted to model the deformation of the interface. Since there are no explicit points or markers to locate the interface, the topology change can be automatically handled. However, the VOF method can form discontinuities at interfaces between grid boundaries when the interface is highly deformed in an unresolved grid, whereas the level-set method may result in unphysical total mass change in a long simulation [11]. Some studies have addressed those issues by modifying the original method [1,5,12] or using a hybrid method [13,11,14]. The front tracking method [15–18] represents the interface using markers

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or points that are connected using Lagrangian interface elements. The interface is tracked and advected based on the fluid velocity field solved on an Eulerian grid. Since the interface is explicitly represented, the front tracking method is considered to be accurate in capturing the moving interface, as well as maintaining the total mass conservation. However, due to its requirement of forming logical connectivities of the interfacial points, the front tracking method raises some numerical difficulties in treating the interface when the topology undergoes large changes such as coalescence or breaking-up. It is more complicated for three dimensional cases.

The beauty of the front tracking method is in its accurate, explicit representation of the interface and a good total mass conservation in a divergence free incompressible flow field. The connectivity of the interfacial points, which increases the complexity of the simulation when the topology undergoes large changes, is actually not necessary. Shin and Juric [19] developed a method to avoid the logical connectivity of the interfacial points by constructing the interface from a level contour. The interface is tracked as done in the original front tracking method. However, the interface is reconstructed periodically by completely discarding the original interfacial points and drawing a new line segment in each interfacial element with a constant indicator connecting the end points. The issue in this method is that the newly constructed interface might not reproduce the original interface when the grid resolution is low or when large deformation occurs. Torres and Brackbill [20] developed a different approach called 'point-set' method to avoid the connectivity. Instead of constructing the indicator field using the normal and surface area of the interface, the indicator field is first constructed and then adjust the constant level contour to coincide with the existing interface using a correction step. Then, the normal and the curvature can be calculated from the indicator field. The topology change is handled by a proper regeneration scheme without the aid of connectivity. Constructing the indicator field and the linear system for the correction step requires each interfacial point to have a *complete* influence domain during the interpolation process. The complete influence domain is not easily obtained for free surface problems that have interfacial points ending at boundaries. The normal and curvature calculations could become exceedingly inaccurate if the interpolation is not complete.

In this paper, the delta function used in a mesh-free reproducing kernel method (RKPM) [21] is chosen for its higher order accuracy and its ability to treat free surfaces. The RKPM interpolation adds a correction function to the original B-spline function so that the reproducing conditions are always satisfied. It also does not require the logical connectivity of the background fluid mesh. This technique offers a more accurate interpolation especially for the calculations of normal and curvature when interfacial points approach to the boundaries. Therefore, we would no longer be limited to model 'closed' interface systems involving bubbles or droplets, but also model and simulate 'open' interface systems such as free surface flows. In addition, non-uniform meshes can also be used to refine mesh locally where higher resolution is needed and deal with complex geometries to achieve more accurate solution. Here, we further improve the proposed algorithm by imposing a simple criterion to smooth out the interface and thus the indicator field so that small disturbances would not induce unreasonable large curvature change. A points regeneration scheme is implemented to automatically treat the topology change by deleting or adding points near contacting surfaces and reconstructing the interface based on a new set of points. This scheme can easily handle complex multiphase flow problems with frequent and large topology changes.

The outline of this paper is as follows. In Section 2, the mathematical model is presented which includes the governing equations for the connectivity-free front tracking method, the construction of the indicator field, the delta function for interpolation using RKPM, and the points regeneration scheme. In Section 3, a standard advection test is performed to verify the mass conservation of this method. Several examples are also presented to show the capability and the accuracy of the algorithm: 2-D oscillating droplet, 2-D dam breaking, 2-D droplet impact and 3-D multi-bubble rising and coalescencing. Finally, the conclusions are drawn in Section 4.

2. Mathematical model and numerical method

2.1. Governing equations for connectivity-free front tracking method

The governing equations for an isothermal multiphase flow can be described using a single set of Navier-Stokes equations with fluid properties varying across the interface. The multi-fluid is treated as 'one-fluid' without the need to handle the jump condition across the interface [22]. The surface tension force can be treated as a singular source term which is added to the momentum equation.

Together, the continuity and momentum equations can be expressed as follows:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} + \mathbf{F}_\sigma \tag{2}$$

where $\mathbf{u}(\mathbf{x}, t)$ is the velocity field, $p(\mathbf{x}, t)$ is the pressure field, ρ and μ are the fluid density and viscosity, respectively, $\mathbf{g}$ is gravity, $\mathbf{F}_\sigma$ is the surface tension force. To evaluate the surface tension, interfacial properties such as unit normal, curvature, and surface area of each interfacial point are required. The traditional front tracking method relies on the connectivity of the interfacial points to obtain these properties. Torres et al.[20] developed a connectivity-free front tracking method, the point-set method, in which they proposed to construct the indicator field first, and the unit normal and curvature are then calcu-