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Adaptive time-step with anisotropic meshing for incompressible flows

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ABSTRACT

This paper presents a method of combining anisotropic mesh adaptation and adaptive time-stepping for Computational Fluid Dynamics (CFD). First, we recall important features of the anisotropic meshing approach using a posteriori estimates relying on the length distribution tensor approach and the associated edge based error analysis. Then we extend the proposed technique to contain adaptive time advancing based on a newly developed time error estimator. The objective of this paper is to show that the combination of time and space anisotropic adaptations with highly stretched elements can be used to compute high Reynolds number flows within reasonable computational and storage costs. In particular, it will be shown that boundary layers, flow detachments and all vortices are well captured automatically by the mesh. The time-step is controlled by the interpolation error and preserves the accuracy of the mesh adapted solution. A Variational MultiScale (VMS) method is employed for the discretization of the Navier–Stokes equations. Numerical solutions of some benchmark problems demonstrate the applicability of the proposed space–time error estimator. An important feature of the proposed method is its conceptual and computational simplicity as it only requires from the user a number of nodes according to which the mesh and the time-steps are automatically adapted.

1. Introduction

Computational Fluid Dynamics simulations have considerably attracted researchers in the last few decades especially with the continuous needs for explanations of the naturally occurring physical flow phenomena that we observe on a daily basis in our surroundings, including pipe flow, flow around airfoils, greenhouse effect and climate predictions, respiratory system and blood circulation, convective heat transfer inside combustion chambers and industrial furnaces and many other applications. The latter phenomena poses deep and complicated scientific problems the resolution of which requires considerable computational resources and long time calculations. Therefore to perform real CFD simulations, accurate solutions within reasonable computational times and costs are highly desirable. Such an objective opens the door to the emergence of numerical methods that aim at optimizing both the spatial and the temporal discretizations [1–6].

There has been a tremendous amount of research designing and implementing methods for space adaptation. In particular, anisotropic mesh adaptation has proved to be a powerful strategy to improve the efficiency of finite element/volume methods, thus reducing storage requirements as well as the computational time. It enables the capture of scale heterogeneities that can appear in numerous physical problems including those having boundary layers, shock waves, edge singularities and moving interfaces [1,7–11].

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In this work, our interest is in time-step adaptation [12–15]. More precisely, in the combination of time and space anisotropic adaptations applied to time-dependant multiphase flows at high Reynolds number. For this purpose, we revisit the theory developed in [1] on anisotropic meshing and modified and extend it to contain an analysis of both space and time interpolation errors. The resulting space–time adaptive algorithm is independent of the properties of the problem at hand and that would significantly lower the computational cost and improve the global accuracy of the calculations. Based on the information given by the derived error estimator in space and the solutions at the previous times, the main idea is to automatically computes an appropriate time-step for the following computations.

Recall that several approaches to build easily unstructured anisotropic adaptive meshes are often based on local modifications [16–19] of an existing mesh. In fact, it mainly requires extending the way to measure lengths following the space directions and can be done by using a metric field to redefine the geometric distances. In parallel, theories about anisotropic a posteriori error estimation (see [20,21]) have been well developed, leading to some standardization of the adaptation process; development of metrics from the analysis of the discretization error and the steering of remeshing by these metrics.

In this paper, we retain the use of a metric constructed directly at the node of the mesh without any direct information from the element, neither considering any underlying interpolation (see [1]). It is performed by introducing a statistical concept: the length distribution function. A second order tensor is used to approximate the distribution of lengths defined by gathering the edges at the node. Using such a technique, we compute the interpolation error along and in the direction of each edge.

The novelty of this paper resides more on implementing a new procedure to compute the metric and the stretching factor. The motivation behind these improvements is to keep only one user parameter, the number of nodes, that links and steers both the space and time adaptation. Moreover, we present an extension of this approach to take into account multicomponent fields (tensors, vectors, scalars). Indeed, rather than considering several metric intersections and thus having much computations to perform for coupled systems, we propose herein an easy way to account for different fields in an a posteriori analysis while producing a single metric field. We will show that the proposed method serves as a powerful tool for approximating multiphase flow problems as it accounts for both the levelset function and all components of the velocity field in only one metric tensor. Finally, we present and detail the implementation aspects to obtain an adaptive meshing under the constraint of a given fixed number of nodes. With such an advantage, we can provide a very useful tool for practical time-dependant incompressible flow problems and avoid a drastic increase in the number of nodes.

The leading idea of this work is then to show that by applying the proposed space–time adaptive algorithm to the recently developed flow solver [22], based on a Variational MultiScale (VMS) method, we are able to produce very good stability and accuracy properties for high Reynolds number flows. In particular, for meshes with highly stretched elements we use an appropriate definition of the stabilization parameters using the directional element diameter.

We aim at testing the performance of the space–time adaptive algorithm on applications that exhibit both spatial heterogeneities and temporal multi-scale phenomena. The first application tests the ability of the developed method to detect different layers of the solution when both the velocity and its gradient are subject to rapid variations. For these reasons, we consider the unsteady Navier–Stokes equations inside the lid driven cavity at different Reynolds numbers and the flow around a circular cylinder benchmark problems. The second type of simulation justifies the ability to reproduce the right evolution of the three-dimensional fluid buckling phenomena. It focuses on free surface and interface problems using a levelset method.

The paper is structured as follows. In Section 2, we introduce the node based metric framework and describe the anisotropic mesh adaptation procedure governed by the length distribution tensor. In Section 3, the interpolation edge error for multicomponent fields is described. Section 4 presents the time interpolation error analysis and the associated time adaptive algorithm. The developed VMS Navier–Stokes solver is outlined in Section 5. Finally, Section 6 provides some numerical results and examples showing the capability of the new highly parallelized space–time anisotropic mesh adaptation.

2. Construction of an anisotropic mesh

In [1], we have developed an a posteriori edge based spatial error estimator relying on the length distribution tensor approach. The anisotropic adaptation involves building a mesh based on a metric map, which means a mesh with edges of unit length for the given metric field. It provides both the size and the stretching of elements in a very condensed information data. Working on a nodal based metric, an anisotropic mesh adaptation procedure is obtained under the constraint of a fixed number of nodes. The details of this technique can be found in [1]. In this section, we retrace the main steps of the construction of this adaptive procedure.

2.1. Edge based error estimation

We consider $u \in \mathcal{C}^2(\Omega) = \mathcal{V}$ and $\mathcal{V}_h$ a simple P^1 finite element approximation space:

$$\mathcal{V}_h = \left\{ w_h \in \mathcal{C}^0(\Omega), w_h|_K \in P^1(K), K \in \mathcal{K} \right\},$$

where $\Omega = \bigcup_{K\in\mathcal{K}} K$ and K is a simplex (segment, triangle, tetrahedron, ...).