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Exponential basis functions in space and time: A meshless method for 2D time dependent problems

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ABSTRACT

In this paper we present a method based on using exponential basis functions (EBFs) to solve well-known two-dimensional time dependent engineering problems such as elasto-dynamic ones. The formulation has much in common with those in three dimensional problems while time is taken as the third axis. The solution is first approximated by a summation of EBFs and then completed by satisfying the time dependent boundary conditions as well as the initial conditions through a collocation method. This is performed by considering a series of spatial and time dependent boundary points to satisfy the boundary conditions through a mixed collocation method. The solution method is presented in a time marching form which is capable of solving variety of problems such as transient heat conduction and wave propagation in solids. Several problems are solved to demonstrate the capabilities of the method.

1. Introduction

Analysis of transient heat conduction and wave propagation problems in two dimensional domains are two research areas which have a wide range of applications in engineering fields. So far, many methods such as the finite difference method (FDM), the finite element method (FEM) or the boundary element method (BEM) have been proposed to solve these problems. Among the aforementioned three methods FDM and FEM directly use space and time discretization and may exhibit sever dispersion effect depending on the approximation used in the solution [1,2]. On the contrary, BEM uses boundary discretization [3], however in time dependent problems finding Green's functions in space and time is not an easy task even for well-known engineering problems. The method has so far been augmented with other numerical schemes to overcome such deficiency [4–9]. It has been experienced that the use of basis functions satisfying the governing equation may reduce the dispersion effect even in case of using FEM (see for instance [10,11]). The idea of using a set of bases satisfying the governing equations has led to emergence of a class of methods known as Trefftz methods. BEM is sometimes categorized in such a class of methods. Another method in this category is the method of fundamental solutions (MFS), basically applied to static problems [12,13]. The reader may refer to [14,15] for the latest developments in MFS and also [16] for a sample of other boundary point methods. Recent advances in this regard can be seen in the application of the method to time dependent problems and again similar features mentioned earlier for BEM are seen in this case (see also [17–19]). On a rather similar basis, one may construct a Trefftz method by a series of exponential basis functions (EBFs) as used in [20]. Such functions are available for a variety of engineering problems defined by governing equations with constant coefficients. The application of the method to various elasto-static problems can be seen in [21–24]. Recent application of the method to non-linear

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problems with moving boundaries can be seen in [25] (see also [26] for the application to problems with fully incompressible materials).

This study is an effort to use EBFs for directly solving 2D time dependent differential equations. The main feature of the presented method is that the solution of the differential equation is expressed as a function in space and time without using the routine schemes such as Laplace transformation or the finite difference method. In this paper we extend the idea used in [20] to solve time dependent problems. We shall focus on problems having application in engineering, such as heat conduction or wave propagation ones. Considering time as an axis, we treat the problems in manner similar to that introduced in [20] while acknowledging the fact that in this case the problems are of initial value type. To this end, the solution of the differential equation is expressed as a series of EBFs. The constant coefficients of the series must be determined from the initial/boundary conditions. Here we satisfy the initial and boundary conditions simultaneously through a collocation scheme using a discrete transformation technique introduced in [20,27,28].

As will be seen later, similar to other methods using MFS, a finite time interval is predefined in the method presented. However, in many practical problems it is needed to consider a rather long period of time for the simulation. We shall introduce a time marching procedure with the aid of the proposed method. To this end, we choose a small time interval and repeat the procedure in a step by step manner while using the information obtained at the end of the time interval as the initial values for the next step.

The layout of the paper is as follows. In the first section the model problem is described. In Section 3 we address the way that the EBFs are found. The solution method is described in Section 4. In Section 5 we introduce the time marching algorithm mentioned earlier. We give a step-by-step summary of the method in Section 6. We show the capability of the method in the solution of five numerical examples in Section 7. Finally in Section 8 we summarize the conclusions made throughout the paper.

2. Model problems

We consider a general 2D time dependent problem, in a vector notation, with the following equation

$$\mathbf{S}^T\mathbf{D}\mathbf{S}\mathbf{u} - c\dot{\mathbf{u}} - \rho\ddot{\mathbf{u}} = \mathbf{0}, \quad (x, y) \in \Omega \text{ and } t \geqslant 0 \tag{1}$$

In the above relation, $\mathbf{u}$ is the vector of main unknown functions while $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are, respectively, its first and second derivatives in time, $\mathbf{S}^T\mathbf{D}\mathbf{S}$ plays the role of the differential operator in which $\mathbf{D}$ is the matrix of material constants. Also c and ρ are two material constants (known as damping and density in elasto-dynamic problems for instance). Moreover Ω is a 2D bounded domain with the boundaries as $\partial\Omega = \Gamma_D \cup \Gamma_N$ while $\Gamma_D \cap \Gamma_N = \emptyset$. The subscripts "$D$" and "$N$" denote Dirichlet and Neumann boundary conditions, respectively. It should be mentioned that we neglect the vector of time dependent body forces in (1). The following generalized boundary conditions are also defined

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_D \tag{2}$$

and

$$\tilde{\mathbf{n}}\mathbf{D}\mathbf{S}\mathbf{u} = \bar{\mathbf{q}} \quad \text{on } \Gamma_N \tag{3}$$

where, $\bar{\mathbf{u}}$ and $\bar{\mathbf{q}}$ are the prescribed time dependent Dirichlet and Neumann boundary conditions and $\tilde{\mathbf{n}}$ is a matrix which contains the components of the outward unit vector normal to the Neumann boundary. The initial conditions of Eq. (1) are given as

$$\mathbf{u}_{|t=0} = \mathbf{I}_1 \tag{4}$$

$$\dot{\mathbf{u}}_{|t=0} = \mathbf{I}_2 \tag{5}$$

where, $\mathbf{I}_1$ and $\mathbf{I}_2$ are the prescribed vectors of the first and second kind of the initial conditions, respectively.

Here it is worthwhile to mention that some of the well-known problems can be defined by appropriately arranging the differential operator $\mathbf{S}$ and the other parameters in (1). For instance, in a transient motion of an elastic system with martial damping, $\mathbf{S}$ is the well-known operator for defining strains as $\boldsymbol{\varepsilon} = \mathbf{S}\mathbf{u}$ while ρ and c are the material density and the material damping, respectively. In that case, $\bar{\mathbf{u}}$ and $\bar{\mathbf{q}}$ in (2) and (3) are the specified displacements and tractions. The matrix of material constants, $\mathbf{D}$, for 2D isotropic elasticity problems can be presented as

$$\mathbf{D} = \begin{bmatrix} D_1 & D_2 & 0 \\ D_2 & D_1 & 0 \\ 0 & 0 & D_3 \end{bmatrix} \tag{6}$$

and $\tilde{\mathbf{n}}$ is defined as

$$\tilde{\mathbf{n}} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix} \tag{7}$$