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Particle-in-cell modeling of relativistic laser–plasma interaction with the adjustable-damping, direct implicit method

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ABSTRACT

Implicit particle-in-cell codes offer advantages over their explicit counterparts in that they suffer weaker stability constraints on the need to resolve the higher frequency modes of the system. This feature may prove particularly valuable for modeling the interaction of high-intensity laser pulses with overcritical plasmas, in the case where the electrostatic modes in the denser regions are of negligible influence on the physical processes under study. To this goal, we have developed the new two-dimensional electromagnetic code ELIXIRS (standing for ELectromagnetic Implicit *X*-dimensional Iterative Relativistic Solver) based on the relativistic extension of the so-called Direct Implicit Method [D. Hewett, A.B. Langdon, Electromagnetic direct implicit plasma simulation, J. Comput. Phys. 72 (1987) 121–155]. Dissipation-free propagation of light waves into vacuum is achieved by an adjustable-damping electromagnetic solver. In the high-density case where the Debye length is not resolved, satisfactory energy conservation is ensured by the use of high-order weight factors. In this paper, we first derive the electromagnetic direct implicit method as a simplified Newton scheme. Its linear properties are then investigated through numerically solving the relation dispersions obtained for both light and plasma waves, accounting for finite space and time steps. Finally, our code is successfully benchmarked against explicit particle-in-cell simulations for two kinds of physical problems: plasma expansion into vacuum and relativistic laser–plasma interaction. In both cases, we will demonstrate the robustness of the implicit solver for crude discretizations, as well as the gains in efficiency which can be realized over standard explicit simulations.

1. Introduction

Particle-in-cell (PIC) codes have become widely used plasma simulation tools owing to their ability to mimic real plasma behavior. Yet the standard PIC algorithm employs an explicit time-differencing, and hence suffers from strict stability constraints on the time step, which needs to resolve the highest-frequency modes of the system [1]. Furthermore, the mesh size must be comparable to the Debye length λ_D in order to prevent the finite-grid instability [1]. As a consequence, explicit PIC codes may find it difficult to cope with the large spatial and temporal scales associated with a number of physical scenarios, thus requiring massively parallel computing facilities [2]. Several alternatives have been developed over the past

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decades to relax these constraints so that the choice of the space and time steps can be dictated by physical accuracy rather than stability conditions. The simplest way to do so is to suppress high-frequency processes within the mathematical model itself. Codes based on the Darwin-field approximation [3,4], gyrokinetic equations [5] or hybrid particle-fluid models [6–10] rely precisely on such an approach. The shortcoming inherent in these codes is the somewhat uncertain domain of validity of their basic assumptions. A second, more involved numerically, possibility retains a fully kinetic and electromagnetic description by using an implicit scheme for the entire Vlasov–Maxwell set of equations. This is the approach dealt with in this work.

The main feature, and difficulty, of a fully implicit PIC scheme is the prediction of the future particles' charge and current densities as functions of the future electromagnetic fields. Two main techniques have been designed to this goal. The first one to be published, the so-called moment method, makes use of the fluid equations to predict future source terms [11–16]. and has been recently extended to the relativistic regime [17]. The present article will focus on the alternate approach, referred to as the direct implicit method, which is based on a direct linearization of the Lorentz equations [18–21]. Most implementations of the direct implicit method start with the so-called D_1 discretization of the Lorentz equation, first presented in Ref. [22]. The relativistic formulation, originally derived in Ref. [23], was implemented, albeit in a simplified form, in the LSP code [24–28].

The direct implicit method proceeds as follows. First, particles' momenta and positions are advanced to an intermediate time level using known fields, yielding predicted charge and current densities. Second, by linearizing the latter quantities around the predicted momenta and positions, we can express correction terms as functions of the future fields and thus derive an implicit wave equation. Once this equation is solved, the particles' quantities are updated. Here we will show that the direct method can be derived as a simplified Newton scheme.

Our main motivation is the simulation of the interaction of an ultra-intense laser pulse with solid-density plasma slabs. The energetic particle beams originating from this interaction stir great interest in many fields spanning inertial confinement fusion [29,26,30–33], high energy density physics [34–37], nuclear physics [38,39] or medical physics [40]. For the high plasma densities considered, the electron plasma frequency ω_p largely exceeds the laser frequency. Using an explicit PIC code, the space and time steps should resolve the high-frequency electron plasma modes of the plasma bulk. However, these modes are of no interest for the problem since they do not affect the laser–plasma interaction nor other potentially important related processes as the subsequent, fast electron-driven ion expansion. By contrast, resorting to an implicit scheme would allow a significantly increased time step, that is, determined only by the need to resolve the incoming laser wave. In this respect, one should realize that the strong wave damping inherent with implicit methods may be harmful in the context of laser–plasma interaction, for which light waves have to travel over many wavelengths. This prompted us to develop an electromagnetic solver with adjustable damping, based on a generalization of the scheme initially proposed by Friedman [41] for the Lorentz equation. We will demonstrate that our adjustable damping scheme tolerates abrupt spatial jumps in the controlling parameter. Our code therefore allows for dissipation-free laser propagation into vacuum, along with strong damping of undesirable plasma waves into the densest part of the target.

Computational efficiency is a major incentive for implementing an implicit method, but the ability of the latter to handle large time steps (i.e., $\omega_p \Delta t \geqslant 2$ and $v_t \Delta t / \Delta x \sim 0.1 - 1$), through which this very efficiency is achieved, also permits to reduce, or even suppress, the aliasing instability responsible for artificial heating in explicit simulations in case of crude spatial discretizations ($\Delta x / \lambda_D \gg 1$) [1]. Yet, the damping associated with the implicit scheme is known to cause nonphysical cooling which may prove detrimental for some applications [1,20,21]. Keeping it at an acceptable level can be achieved, as will be shown, by increasing the order of the weight functions, which, by weakening the aliasing instability [42,43], allows to limit the level of damping required to achieve satisfactory energy conservation.

The paper is organized as follows. In Section 2, we recall the basic principles of the PIC technique, give the implicit time-discretized equations to solve, and derive within a simplified Newton formalism the relativistic direct implicit method. In Section 3, we outline the numerical resolution of the wave equation as implemented in our newly developed, 2Dx–3Dv code ELIXIRS (ELectromagnetic Implicit *X*-dimensional Iterative Relativistic Solver). The introduction of implicit injecting/outgoing boundary conditions for the electromagnetic field is also discussed. Section 4 is devoted to the linear properties of the direct implicit method through the resolution of the electromagnetic and electrostatic dispersion relations. The effects of finite space and time steps, adjustable damping and high-order weight factors will be accounted for. Finally, in Section 5, our code is benchmarked against explicit simulations for two kinds of physical problems: the expansion of a plasma slab in vacuum, and the interaction of an ultra-intense laser pulse with an overcritical plasma target. The sensitivity of the simulation results to the damping parameter and the number of macroparticules will be addressed.

2. The relativistic direct implicit method as a simplified Newton scheme

In contrast to Ref. [23], we present here a derivation of the electromagnetic direct implicit method for the relativistic case within a Newton iterative scheme and a weak formulation of Maxwell's equations. Note that a similar iterative algorithm was originally proposed in the non-relativistic electrostatic case in Ref. [19]. Anticipating our need of a dissipation-free propagation of light waves inside the vacuum region of the simulation domain, we introduce a generalization of the adjustable damping scheme proposed and used in the electrostatic regime by Friedman [41].