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Solution of time-convolutionary Maxwell's equations using parameter-dependent Krylov subspace reduction

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ABSTRACT

We suggest a new algorithm for the solution of the time domain Maxwell equations in dispersive media. After spacial discretization we obtain a large system of time-convolution equations. Then this system is projected onto a small subspace consisting of the Laplace domain solutions for a preselected set of Laplace parameters. This approach is a generalization of the rational Krylov subspace approach for the solution of non-dispersive Maxwell's systems. We show that the projected system preserves such properties of the initial system as stability and passivity. As an example we consider the 3D quasistationary induced polarization problem with the Cole–Cole conductivity model important for geophysical oil exploration. Our numerical experiments show that the introduction of the induced polarization does not have significant effect on convergence.

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1. Introduction

It was observed since 1920s [29], that electrical conductivity and dielectric permeability of rocks are frequency dependent. This phenomenon has generic name the Dispersive Media, or the Induced Polarization for the frequency-dependent conductivity, and intensively described in physical and geophysical literature [5–7,18,22,25,26,37].

The time-domain evolution of electromagnetic fields in dispersive medium is described by convolutionary Maxwell's system. There are two approaches to solving such problems. One way is time-stepping [23,24,34]. Even for the non-dispersive (a.k.a. inductive) diffusive problems this approach can be expensive due to stability limitations and slow convergence. However, the computation of convolution operator requires additional several time layers, that increases the cost compared to the non-dispersive problem. Another approach is to transform the frequency domain solution to the time domain using the discrete inverse Fourier transform [12,20]. To evaluate such integrals one needs to compute a frequency domain solution for every quadrature node that may require hundreds forward solutions for good enough accuracy. We should point out, that some improvement in this direction could be achieved by using recently developed optimized quadratures on complex plane [31].

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Krylov subspace methods (originally intended for spectral problems and linear algebraic systems) became an increasedly popular tool for the solution of parabolic equations (the matrix exponential function) in 1980s [14,16,21,28,32]. They directly project semi-discrete systems onto Krylov subspaces and can be considered as spectrally optimized explicit time-stepping methods. Such an approach (a.k.a. spectral Lanczos decomposition method) was efficiently applied to the diffusive (non-dispersive) Maxwell system in [8]. It yields faster than exponential (quadratic) convergence rate, however it is significantly affected by the problem stiffness. A more recent approach, based on so-called rational Krylov subspaces (RKS) introduced in [27], allows one to circumvent this drawback. The RKS reduction (RKSR) (a.k.a. the RKS projection) method was applied to inductive diffusive Maxwell's equations in [4,9,11,17] (see also some important results on the RKSR analysis in [3]). Similar to the implicit time-stepping and the contour integration methods the RKSR requires the solution of a number of shifted frequency domain problems, but in the latter method this number is much smaller thanks to Galerkin's optimal properties.

Due to nonlinear frequency dependence of the operator neither Krylov nor rational Krylov subspace method can be directly applied to the dispersive problems. In this work we develop a concept related in some sense to parametric model reduction [33], i.e, we project frequency domain problem onto a so-called parameter-dependent Krylov subspace (PDKS) with properly chosen parameters. We call this method the parameter-dependent Krylov subspace reduction (PDKSR). This approach is also related to so-called nonlinear Krylov subspaces, used for the computation of nonlinear spectrum [30]. For the non-dispersive case it is equivalent to the RKSR. The PDKSR is first applied in the frequency domain and then the obtained solution transformed to the time domain using numerical integration. We show that the projected problem preserves such important properties of the initial problem as stability and passivity.

As an example of application, we consider the Cole–Cole induced polarization model in conductive media (corresponding to a fractional order PDE system) arising in electromagnetic geophysical exploration. Although the conductivity is complex in this case, its high frequency asymptote is the same as for non-dispersive case. Hence for the parameter-dependent Krylov Subspace we use the real poles optimized for large-scale inductive problems in [9]. The numerical experiment shows that the introduction of the Cole–Cole dispersion qualitatively changes behavior of the solution, however it has very little effect on convergence speed of the subspace reduction.

2. Formulation of the problem

To fix the idea, we consider the quasistationary time-domain Maxwell system in $\mathbf{R}^3 \times \mathbf{R}$

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J}_c + \mathbf{J}', \\ \nabla \times \mathbf{E} &= -\mu \tfrac{\partial \mathbf{H}}{\partial t}. \end{aligned} \tag{1}$$

Here $\mathbf{H}$ is magnetic field, $\mathbf{E}$ is electric field, $\mathbf{J}_c, \mathbf{J}'$ are conductivity and exciting (external) currents respectively; both currents are real functions of time $t \in \mathbf{R}$ and space $\mathbf{r} \in \mathbf{R}^3$ coordinates. We assume for simplicity that magnetic permeability $\mu > 0$ is constant. The quasistationary approximation (when the displacement current is neglected) is commonly assumed in deep geophysical electromagnetic exploration [6].

The conductivity current $\mathbf{J}_c$ can be expressed with the help of the generalized Ohm law in terms of electric field as follows:

$$\mathbf{J}_c = \sigma * \mathbf{E} = \int_{-\infty}^{\infty} \sigma(\tau)\mathbf{E}(t-\tau)d\tau, \tag{2}$$

where σ is an electric conductivity function of space and convolutional time τ, and

$$\sigma = 0, \quad \text{for } \tau < 0.$$

We impose the initial condition

$$\mathbf{E} = 0, \quad \mathbf{H} = 0, \quad \text{for } t < 0, \tag{3}$$

and for consistency we require that $\mathbf{J}'|_{t<0} = 0$.

Remark 1. The displacement current

$$\mathbf{J}_d = \epsilon * \frac{\partial}{\partial t}\mathbf{E} = \int_{-\infty}^{\infty} \epsilon(\tau)\frac{\partial \mathbf{E}(t-\tau)}{dt}d\tau,$$

with dispersive dielectric permittivity function ϵ can be added to the convolution system (2) without changing its structure by substituting

$$g = \sigma - \frac{d\epsilon}{d\tau}$$

instead of σ in (2), so the algorithm described below is applicable for the case of dispersive dielectric permittivity too (possibly with a different choice of optimized interpolation frequencies).

We also need the following assumptions on the temporal growth of the coefficient and $\mathbf{J}'(t, \mathbf{r})$.

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