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A direct comparison between volume and surface tracking methods with a boundary-fitted coordinate transformation and third-order upwinding

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Abstract

A Volume Tracking (VT) and a Front Tracking (FT) algorithm are implemented and compared for locating the interface between two immiscible, incompressible, Newtonian fluids in a tube with a periodically varying, circular cross-section. Initially, the fluids are stationary and stratified in an axisymmetric arrangement so that one is around the axis of the tube (core fluid) and the other one surrounds it (annular fluid). A constant pressure gradient sets them in motion. With both VT and FT, a boundary-fitted coordinate transformation is applied and appropriate modifications are made to adopt either method in this geometry. The surface tension force is approximated using the continuous surface force method. All terms appearing in the continuity and momentum equations are approximated using centered finite differences in space and one-sided forward finite differences in time. In each time step, the incompressibility condition is enforced by a transformed Poisson equation, which is linear in pressure. This equation is solved by either direct LU decomposition or a Multigrid iterative solver. When the two fluids have the same density, the former method is about 3.5 times faster, but when they do not, the Multigrid solver is as much as 10 times faster than the LU decomposition. When the interface does not break and the Reynolds number remains small, the accuracy and rates of convergence of VT and FT are comparable. The well-known failure of centered finite differences arises as the Reynolds number increases and leads to non-physical oscillations in the interface and failure of both methods to converge with mesh refinement. These problems are resolved and computations with Reynolds as large as 500 converged by approximating the convective terms in the momentum equations by third-order upwind differences using Lagrangian Polynomials. When the volume of the core fluid or the Weber number decrease, increasing the importance of interfacial tension and leading to breakup of the interface forming a drop of core fluid, the FT method converges faster with mesh refinement than the VT method and upwinding may be required. Finally, examining the generation of spurious currents around a stationary "bubble" in the tube for Ohnesorge numbers between 0.1 and 10 it is found that the maximum velocity remains approximately the same in spite mesh refinements when VT is applied, whereas it is of the same order of magnitude for the coarsest mesh and monotonically decreases with mesh refinement when FT is applied.

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1. Introduction

Flows of two or more immiscible fluids exhibit numerous phenomena with intrinsic scientific interest and important practical applications. Such applications from the oil and plastics industries include the production of oil from the ground (flow with water or steam in porous media), its water-facilitated transportation through pipelines, its chemical transformation in packed-bed reactors (flow with gaseous reactants in porous media) and the production of bicomponent and composite plastics and layered films (flow inside and out of channels). The overall applicability and performance of operations involving flows of two fluids are strongly dependent on their interface shapes. So, we must develop an understanding of the effects of the bulk properties and flow conditions on the interface evolution. In turn, a difficulty in developing reliable theoretical models for flows involving more than one fluid lies in the non-uniqueness of solutions, which is intimately related to the interface shape and its evolution. This strong coupling between the interface deformation and bulk motion necessitates computing them simultaneously. Our motivation for studying the two-phase flow in a circular tube with sinusoidal variation in its cross-section is that this is the simplest of the shearing flows of two fluids, which can adequately approximate the flow that takes place in a packed-bed reactor and, more generally in porous media. This model geometry of the tube wall has been used extensively for simulating single-phase flow of both Newtonian [1,2] and non-Newtonian fluids [3] and, although it cannot describe latitudinal dispersion in an actual packed-bed or exchange of fluids between nearby conduits, it simulates the converging–diverging character of the flow. In such reactors inertial, viscous and capillary forces are all comparable in magnitude and none can be neglected *a priori*. The fundamental mechanisms that give rise to the different flow regimes are rooted at the scale of the pore, where the two-phases flow and compete for the available space. Among the flow regimes that have been observed experimentally in a packed-bed reactor are: (a) The trickling regime, in which both phases are continuous, (b) The pulsing regime, in which one phase forms large discontinuous slugs within the other one which remains continuous, and (c) The spray and the bubbling regimes, in which the dispersed phase forms drops or bubbles much smaller than the typical constriction, which are surrounded by the continuous phase.

The numerical simulations of two-phase flows involve the solution of the Navier–Stokes equations for the two fluids with specified boundary and interfacial conditions. Several approaches to solve these moving interface problems are available in the literature, which have their own advantages and disadvantages. In particular, the free surface can be dealt with either an Eulerian or a Lagrangian numerical approach. The Eulerian approach involves the use of a discretization mesh that remains fixed in space with both fluids moving through it, while in the Lagrangian approach the mesh is convected by the flow in an effort to follow the moving interface and, thus, only simple nonintersecting interfaces can be represented due to limitations on the amount of mesh distortion allowed. Most finite element as well as spectral methods are developed around a Lagrangian approach and have been used successfully in calculating time dependent solutions of both single [4] and two-phase flows [5–8] in which free surfaces are involved. More recently, we advanced a mesh generation method based on solving a set of elliptic differential equations that can follow very large interface distortions, but still additional programming effort is needed to allow for interface breaking and merging [9,10]. Because we are further interested in readily predicting the above mentioned flow regimes and how they evolve, using either FEM or spectral methods in combination with a Lagrangian approach is precluded, since they cannot, at the present time, model such complex breaking and reforming interfaces [11].

In this paper, we utilize two basic Eulerian approaches for computing the evolving interface, namely a Volume Tracking (VT) which captures the interface after calculating the velocity field at each time step and a Front Tracking (FT) method, which explicitly advances the interface by following the motion of Lagrangian particles placed on it, [11]. In both methods the bulk fluids are treated as a single fluid properly accounting for the different fluid properties in each phase and the entire domain is partitioned into stationary computational cells irrespective of the changing topology of each phase, while interfacial tension is incorporated as a body force in the momentum balances multiplied by a delta function. In VT a marker function is introduced in