Journal of Computational Physics 229 (2010) 32–57

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Improving filtering and prediction of spatially extended turbulent systems with model errors through stochastic parameter estimation

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ARTICLE INFO

Article history:
Received 13 April 2009
Received in revised form 9 September 2009
Accepted 18 September 2009
Available online 29 September 2009

Keywords:
Stochastic parameter estimation
Kalman filter
Filtering turbulence
Data assimilation
Model error
Predictability

ABSTRACT

The filtering and predictive skill for turbulent signals is often limited by the lack of information about the true dynamics of the system and by our inability to resolve the assumed dynamics with sufficiently high resolution using the current computing power. The standard approach is to use a simple yet rich family of constant parameters to account for model errors through parameterization. This approach can have significant skill by fitting the parameters to some statistical feature of the true signal; however in the context of real-time prediction, such a strategy performs poorly when intermittent transitions to instability occur. Alternatively, we need a set of dynamic parameters. One strategy for estimating parameters on the fly is a stochastic parameter estimation through partial observations of the true signal. In this paper, we extend our newly developed stochastic parameter estimation strategy, the Stochastic Parameterization Extended Kalman Filter (SPEKF), to filtering sparsely observed spatially extended turbulent systems which exhibit abrupt stability transition from time to time despite a stable average behavior. For our primary numerical example, we consider a turbulent system of externally forced barotropic Rossby waves with instability introduced through intermittent negative damping. We find high filtering skill of SPEKF applied to this toy model even in the case of very sparse observations (with only 15 out of the 105 grid points observed) and with unspecified external forcing and damping. Additive and multiplicative bias corrections are used to learn the unknown features of the true dynamics from observations. We also present a comprehensive study of predictive skill in the one-mode context including the robustness toward variation of stochastic parameters, imperfect initial conditions and finite ensemble effect. Furthermore, the proposed stochastic parameter estimation scheme applied to the same spatially extended Rossby wave system demonstrates high predictive skill, comparable with the skill of the perfect model for a duration of many eddy turnover times especially in the unstable regime.

1. Introduction

Filtering is the process of obtaining the best statistical estimate of a physical system from partial observations of the true signal. In many contemporary applications in science and engineering, real-time filtering of a turbulent signal involving many degrees of freedom is needed to make accurate predictions of the future state. Important practical examples involve

DOI of original article: 10.1016/j.jcp.2009.08.019

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doi:10.1016/j.jcp.2009.09.022

the real-time filtering and prediction of weather and climate as well as the spread of hazardous plumes or pollutants. A major difficulty in accurate filtering and prediction of noisy turbulent signals with many degrees of freedom is model error [35]: the fact that the true signal is processed through an imperfect model where important physical processes are parameterized due to inadequate numerical resolution or incomplete physical understanding. Under these circumstances, it is natural to devise strategies for parameter estimation to cope with model errors to improve both filtering and prediction skill [32,2–4,9,8,6,17].

Recently, we proposed the use of the test models with stochastic parameter estimation [14], where certain parameters of the system such as damping and external forcing can be learned from the observations. This approach, called the Stochastic Parameterization Extended Kalman Filter (SPEKF) in [14], has been shown to be effective when applied to a single mode [14]. In this paper, we extend this stochastic parameterization strategy to filtering sparsely observed spatially extended systems with a decaying (Kolmogorov type) turbulent spectrum [19] and intrinsic model errors as in [14]. According to results in [19] for the perfectly specified filter case, we can obtain accurate filtered solutions by filtering only the observed modes with a reduced filter when the energy spectrum decays as a function of wavenumber. Our new approach here is to unify ideas from these two papers [19,14], that is, to apply SPEKF only to the observed modes and let the estimates of the remaining unobserved and least energetic modes to be unfiltered and propagated in time with the climatological model.

In this paper, we will test our newly developed strategy on a nontrivial toy model for turbulent barotropic Rossby waves in a one-dimensional periodic domain with a time periodic external forcing. We design this model such that it exhibits instability that mimics the baroclinic instabilities in the midlatitude atmosphere at random times despite a stable long time average behavior. In this example, the model errors are introduced through our lack of information about the onset time and duration of instability regimes. Moreover, we also introduce a second source of model error by purposely not specifying the external forcing. In our numerical experiments with this example, we resolve this model with 105 equally spaced grid points in a periodic domain and consider rather sparse observations at 15 equally spaced grid points. Through extensive numerical studies, we will find that our new stochastic parametrization strategy, especially the one that combines both multiplicative and additive bias corrections, produces a significantly improved and robust filtering skill as was shown earlier in the one-mode context in [14]. We will also find significant improvement in predictive skill with the combined model relative to the mean model.

The paper is organized as follows. In Section 2, we describe idealized spatially extended turbulent systems: this includes the simplest stochastic models for turbulent signals, the turbulent Rossby waves problem as an example, and the two-state Markov process as an underlying mechanism for stability regime transitions. In Section 3, we discuss various strategies for filtering with model errors, including the standard approaches such as the mean stochastic model (MSM) and the standard online bias correction strategy using extended Kalman filter [22,1,7] which motivates the Stochastic Parameterization Extended Kalman Filter (SPEKF) in our recent work [14]. In this section, we also review results of SPEKF in the one-mode context [14] and then specify a strategy for implementing SPEKF for spatially extended systems with sparse observations. In Section 4, we present the numerical results for filtering, including the correctly specified and unspecified forcing and the filter skill robustness throughout variations of parameters. In Section 5, we extensively study the predictive skill on one Fourier mode in a "super-ensemble" setup. In particular, we try to understand the effect of errors from initial conditions and a finite ensemble size. Consequently, we show results on the full SPDE. We close both Sections 4 and 5 with short summaries. We end the paper with concluding discussions in Section 6. Detailed calculations of correlation function and Kalman filter formulas are reported in Appendixes A and B.

2. Idealized spatially extended turbulent systems

The simplest models for representing turbulent fluctuations involve replacing nonlinear interaction by additional linear damping and stochastic white noise forcing in time which incorporate the observed climatological spectrum and turnover time for the turbulent field [10,28,30].

As in the standard classical numerical analysis test criteria for finite difference schemes [33,29], we start with a linearized complex $s \times s$ PDE at a constant coefficient background, $\vec{u}_t = \mathcal{P}(\partial_x)\vec{u} + \vec{f}(x,t)$. Here $\vec{f}(x,t)$ is a known deterministic forcing term. In accordance with the above approximations, additional damping $-\gamma(\partial_x)\vec{u}$ and spatially correlated noise

$$\sigma(x)\dot{W}(t) \equiv \sum_{k=-\infty}^{\infty} \sigma_k \dot{W}_k(t) e^{ikx}, \tag{1}$$

where $W_k(t)$ are independent complex Wiener processes for each $k \geqslant 0$ and the independent real and imaginary parts have the same variance 1/2 and $\sigma_{-k} = \sigma_k$ and $W_{-k}(t) = W_k^*(t)$, are added to the PDE to represent the small scale unresolved turbulent motions resulting in the basic frozen coefficient canonical PDE. For simplicity in notation here we discuss a scalar field in a single space real variable but everything generalizes to a matrix system of stochastic PDEs in several space dimensions.

2.1. The simplest stochastic models for turbulent signals

With the above motivation, we consider solutions of the real valued scalar stochastic partial differential equation (SPDE):