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New absorbing layers conditions for short water waves

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ABSTRACT

We develop a new PML formulation for the linearized shallow-water equations including the Coriolis force. The construction process is based on the uncoupling of the velocity components with the depth of water. Then the damping effect is only applied to the propagative modes just as was formerly done by Nataf [1] to the linearized Euler equations to enforce the long-time stability. We assess numerically the performance of the new absorbing condition and we illustrate in particular that it is stable for long-time simulations.

1. Introduction

Many applications require to solve numerically dispersive wave problems in a domain which is much smaller than the physical one. A current method consists in applying a local absorbing boundary condition (ABC) on the exterior boundary which is used to limit the computational domain. The ABC should minimize spurious reflections when waves impinge on the exterior boundary which has no physical meaning. Obviously, the more efficient the ABC is, the more accurate the numerical solution will be. A lot of works have been dealing with the construction of ABCs that after discretization lead to a stable and accurate scheme and we refer to [2,3] for discussion on related issues and for reviews on the subject. Most of the ABCs have been designed for either time-harmonic waves or for non-dispersive time-dependent waves and the use of ABCs in case of dispersive waves is much more difficult. A important example where dispersive waves must be considered is that of meteorological models which take into account the Earth's rotation [4]. Recently, Joolen et al. [5] have developed a new numerical scheme including high-order ABCs for dispersive waves which are based on Higdon's ABCs [6]. While it is possible to construct local ABCs easy to implement, their efficiency to damp spurious reflections often suffer from the corner problem. This difficulty can be overcome by using Perfectly Matched Layers just as was suggested by Bérenger [7,8]. The very attractive property of the PML is it absorbs all the waves without spurious reflection and the corner problem is easily solved by a suitable fit of the layer parameters. Moreover, the coupling of the physical system with the PML condition is easy to handle numerically. Many works have been devoted to the design of PML for various applications and as far as the shallow-water equations are concerned, Navon et al. [9] have recently developed a split Perfectly Matched Layer (PML) scheme for the linearized system which is based on an explicit finite-difference discretization scheme. This PML requires a stabilization process involving a 9-point Laplacian filter to avoid the split PML supports unstable solutions. This question was for-

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merly addressed for the linearized Euler equations in [10] and next in [11] for more general flows. Both in [10] and [11], the PML is obtained via a change of coordinate in the complex plane applied to the direction normal to the boundary. This amounts to replacing all the normal-derivatives in the Euler system by an operator which is still differential in the normal-direction but pseudo-differential in the other variables. In [1], Nataf has proposed another strategy which leads to the design of a stable PML for the Euler equation. The idea consists in applying the Smith factorization to the Euler equations in order to uncouple the propagative part of the solution from the transport one. Then the PML is constructed in such a way that only the modes that could produce reflections are damped. Thus the vorticity modes, which satisfy transparent conditions [12] on the computational boundary domain, are not damped and the resulting scheme seems to be stable as numerical experiments show.

In this paper, we propose a new PML formulation of the linearized shallow-water equations whose construction is based on the splitting of the primal system. Under the assumption the Coriolis force is constant, the depth h can be uncoupled from the velocity components by applying some elementary combinations which preserve the differential structure of the initial equations and the resulting formulation involves now a Klein–Gordon equation. The decomposition results in uncoupling the advective part of the wave from the vorticity (Section 2). After having remarked that the vorticity waves can be absorbed via an appropriate transparent condition and therefore do not need to be damped in the artificial layer, the PML condition is written by applying a complex coordinate change to the advective unknown only (Section 3). In Section 4 we discuss the practical handling of the method and we present the numerical scheme used to discretize the PML equations. Numerical results are presented in section 5. They confirm the new layer is perfectly matched to the physical domain and by considering long times of simulation, they seem to illustrate the long-time stability of the PML system.

2. Setting of the primal system

The shallow-water model contains some of the important dynamical features of the atmosphere and ocean and experience has shown that it is capable of describing main aspects of their motions. Let us consider a fluid with constant and uniform density. The height of the fluid surface above the reference level $z = 0$ is $h := h(t, x, y)$. Even if h varies in space and time, we suppose that we can choose a characteristic value for the depth which is denoted by H. In the same way, we assume there exists a characteristic horizontal length scale for the motion which we call L. Then the fundamental condition which characterizes shallow-water theory is

$$\delta = \frac{H}{L} \ll 1. \tag{1}$$

We suppose the rotation axis of the fluid coincides with the z-axis so that the Coriolis parameter f is a constant. The velocity has components u, v and w parallel to the x-, y- and z-axis, respectively. The pressure of the fluid can be arbitrarily imposed and herein, we take it to be constant which implies in particular that the horizontal pressure gradient is independent of z. It is therefore consistent to assume that the horizontal velocities themselves are z-independent if they are at $z = 0$. Moreover, w can be uncoupled from u and v, observing that this simplification is no more possible if the density varies with z. At last, the fluid is supposed to be inviscid, which amounts to neglect viscosity effects. According to [4], the linearized horizontal momentum equations read then as:

$$\begin{cases} Gh + H(\partial_x u + \partial_y v) = 0, \\ Gu + \partial_x h - f v = 0, \\ Gv + \partial_y h + fu = 0, \end{cases} \tag{2}$$

where G denotes the transport operator defined by:

$$G = \partial_t + U\partial_x + V\partial_y,$$

and U, V and H are positive constants which characterize the equilibrium state around which the linearization process has been done. In this paper, we consider the subsonic case corresponding to $U^2 + V^2 < H$.

Under the assumption f is constant, the unknown h can be uncoupled from the two other ones by using some combinations of the three equations. Indeed, by deriving the second (resp. third) equation of (2) with respect to x (resp. y), and by summing up the two resulting equations, we obtain:

$$G(\partial_x u + \partial_y v) = -\Delta h + f(\partial_x v - \partial_y u). \tag{3}$$

Then by applying G to the first equation and by plugging (3) into the resulting equation, we have:

$$(G^2 - H\Delta)h + Hf(\partial_x v - \partial_y u) = 0. \tag{4}$$

Next, by deriving the second (resp. third) equation of (2) with respect to y (resp. x), we get a writing of $G(\partial_x v - \partial_y u)$ which can be used after applying G to (4). We thus get that h is solution to the third-order differential equation:

$$(G^2 - H\Delta)(Gh) + f^2(Gh) = 0, \tag{5}$$