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Linearized reduced-order models for subsurface flow simulation

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ABSTRACT

A trajectory piecewise linearization (TPWL) procedure for the reduced-order modeling of two-phase flow in subsurface formations is developed and applied. The method represents new pressure and saturation states using linear expansions around states previously simulated and saved during a series of preprocessing training runs. The linearized representation is projected into a low-dimensional space, with the projection matrix constructed through proper orthogonal decomposition of the states determined during the training runs. The TPWL model is applied to two example problems, containing 24,000 and 79,200 grid blocks, which are characterized by heterogeneous permeability descriptions. Extensive test simulations are performed for both models. It is shown that the TPWL model provides accurate results when the controls (bottom hole pressures of the production wells in this case) applied in test simulations are within the general range of the controls applied in the training runs, even though the well pressure schedules for the test runs can differ significantly from those of the training runs. This indicates that the TPWL model displays a reasonable degree of robustness. Runtime speedups using the procedure are very significant–a factor of 100–2000 (depending on model size and whether or not mass balance error is computed at every time step) for the cases considered. The preprocessing overhead required by the TPWL procedure is the equivalent of about four high-fidelity simulations. Finally, the TPWL procedure is applied to a computationally demanding multiobjective optimization problem, for which the Pareto front is determined. Limited high-fidelity simulations demonstrate the accuracy and applicability of TPWL for this optimization. Future work should focus on error estimation and on stabilizing the method for large models with significant density differences between phases.

1. Introduction

The development of reduced-order modeling procedures has received significant attention in recent years. Reduced-order models (ROMs) are particularly well suited for use in minimization procedures, where the forward model must be run many times (e.g., hundreds or thousands) for the determination of optimal design or operating parameters. Our particular interest here is in subsurface flow problems, specifically oil reservoir simulation. Other closely related applications include aquifer management and the geological storage of carbon dioxide.

A number of the previous ROM procedures developed for subsurface flow applications are based on the use of proper orthogonal decomposition (POD). This entails performing one or more high-fidelity training simulations, saving snapshots (state vectors) at a number of time steps from these simulations, and then constructing a set of basis functions from these snapshots. The basis functions are generated through a singular value decomposition of the snapshot matrix, which is analogous to

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an eigen-decomposition of the covariance matrix formed from the snapshots. Efficiency is achieved because only relatively few basis functions must be retained; i.e., flow solutions can be represented in terms of just the leading basis vectors.

POD procedures were first introduced by Lumley [1] to identify coherent structures in dynamical systems and have subsequently been applied in a number of application areas (e.g., [2–7]). Vermeulen et al. [8] appear to be the first to have applied POD techniques within the context of subsurface flow problems. They achieved substantial speedups for groundwater flow involving a single (water) component. In this case, however, the governing equation is linear (or nearly linear). Jansen and coworkers [9,10] applied POD techniques to two-phase (oil–water) flow. Here, due to differing phase viscosities and phase interference (relative permeability) effects, the problem is nonlinear and the reported speedups were much more modest; e.g., a factor of 3 or less.

In recent work targeting oil–water reservoir simulation problems, we incorporated new procedures designed to enhance the basic POD approach (Cardoso et al. [11]). These included a snapshot clustering procedure [12] and a missing point estimation (MPE) technique [13]. MPE acts to eliminate rows from the basis matrix and thus accelerates the requisite matrix–matrix multiplications. We incorporated the POD techniques into a general purpose research simulator and applied them to geologically realistic models containing 60,000 grid blocks. Speedups of up to about a factor of 10 were achieved.

There are however several limitations that impact the degree of speedup attainable from standard POD procedures for this nonlinear problem. Consider a system containing N_c grid blocks (which corresponds to $2N_c$ unknowns for the oil–water problem) and suppose that this system can be represented using ℓ basis functions. For specificity, in an example in our earlier study [11], $N_c = 60{,}000$ and $\ell = 40$. The standard POD procedure works at the level of the linear solver, reducing the sparse $2N_c \times 2N_c$ linear system to a full $\ell \times \ell$ system. Solver time is thus reduced substantially. Computational requirements for other operations, by contrast, such as the construction of the Jacobian matrix (which must be performed at each iteration of every time step) are not reduced at all through the use of the standard POD technique. The application of MPE procedures does eliminate the need for some rows of the Jacobian matrix (and thus they need not be constructed), but the required number of rows in the Jacobian would still be expected to be a reasonable fraction of N_c. In addition, a full implementation of MPE requires modifications throughout the simulator. This may be complicated if the MPE procedure is incorporated into a comprehensive general purpose simulator.

We also observed that standard POD procedures are less robust and/or require more basis functions as the problem becomes more nonlinear, as occurs with strong gravitational effects and with highly nonlinear relative permeability functions [11]. In these cases, the speedups offered by standard POD techniques will be quite modest at best.

Recent linearization procedures, specifically the trajectory piecewise linearization (TPWL) introduced by Rewienski and White [14,15], appear to offer a means for addressing some of the limitations in standard POD techniques highlighted above. The TPWL approach combines reduced-order modeling with linearization of the governing equations. For the implementation described in this paper, TPWL proceeds as follows. First, two training simulations are performed, from which the states and converged Jacobian matrices are saved. A basis matrix is then constructed from the states using proper orthogonal decomposition, and this basis is used to form reduced states and reduced Jacobian matrices. In subsequent simulations, new states are represented in terms of linear expansions around previously simulated (and saved) states and Jacobians. Very high degrees of computational efficiency are achieved because all of these computations are performed in reduced space. TPWL has been successfully applied in a number of application areas, including computational fluid dynamics [16], heat-transfer modeling [17], biomedical micro-electromechanical systems (BioMEMS) [18], electronic circuits [19,20], and moving electromagnetic devices [21]. Stabilized TPWL methods have also been recently presented by, e.g., [22,23]. TPWL procedures do not, however, appear to have been considered for oil reservoir simulation or for any closely related subsurface flow applications.

Our goal in this paper is to develop and apply TPWL procedures for subsurface flow problems. Toward this end we formulate a TPWL representation for oil–water flow and present extensive results for challenging subsurface flow problems. Our examples involve highly heterogeneous geological models and include different phase densities (in one case) and nonlinear relative permeabilities. Numerical results demonstrate that, for test simulations that are within the general range of the training runs, the TPWL procedure can provide predictions in close agreement with high-fidelity simulations with runtime speedups of a factor of 100–2000. The computational overhead required to construct the TPWL model is the equivalent of about four high-fidelity simulations.

This paper proceeds as follows. We first present the governing equations for oil–water flow and develop the linearized representation. POD procedures and their use in conjunction with the linearized oil–water model are then described. Next, some implementation issues are discussed and the overall TPWL algorithms are provided. We then present simulation results for two cases, containing 24,000 and 79,200 grid blocks. Accurate results for oil and water production and water injection rates are achieved for both cases for a wide variety of well control schedules. We additionally apply the TPWL representation for a multiobjective optimization problem and demonstrate close correspondence with selected high-fidelity computations. Finally, additional issues and outstanding challenges are discussed and concluding remarks are presented.

2. Trajectory piecewise linearization (TPWL) for subsurface flow

In this section we present the equations for two-phase subsurface flow and briefly describe the basic finite volume discretization. The TPWL procedure is then developed in detail. This includes a short description of the construction of the POD basis matrix, which is required for the TPWL representation. We note that there are several heuristic components in our TPWL formulation, which are discussed in the development that follows.