Journal of Computational Physics 229 (2010) 851–874

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Improved transmission conditions for a one-dimensional domain decomposition method applied to the solution of the Helmholtz equation

Bruno Stupfel *

CEA, DAM, CESTA, F-33114 Le Barp, France

ARTICLE INFO

Article history:
Received 23 January 2009
Received in revised form 15 September 2009
Accepted 10 October 2009
Available online 20 October 2009

Keywords:
Electromagnetism
Helmholtz equation
Frequency domain
Domain decomposition method
Transmission conditions
Numerical methods

ABSTRACT

The scattering problem of a time-harmonic electromagnetic wave from a perfect electric conductor (PEC) coated with materials is considered, and solved by coupling a finite element method with an integral equation prescribed on the outer boundary of the computational domain. To reduce the numerical complexity, a one-dimensional domain decomposition method (DDM) is employed: the computational domain is partitioned into concentric subdomains (SDs), and Robin transmission conditions (TCs) are prescribed on the interfaces. For some configurations and/or materials, the convergence of the corresponding DDM algorithm happens to be slow. A possible remedy is to enhance the efficiency of the TCs by approximating the exact ones more accurately. To this end, we first consider the simplified 2D problem of a circular PEC cylinder with an homogeneous coating and up to four SDs with circular interfaces, thus allowing to obtain the exact TCs in closed-form. Approximate local or non-local TCs are derived from these exact ones, and numerical examples demonstrate their superiority over the standard Robin TCs. Then, the case of an elliptical PEC cylinder with one interface in free-space is investigated. Also, the issues pertaining to the uniqueness of the solutions and convergence of the algorithm are addressed.

1. Introduction

The scattering problem of a time-harmonic electromagnetic wave from complex 3D inhomogeneous objects embedded in free-space can be accurately solved by coupling a finite element method (FEM) with an integral equation (IE) prescribed on the outer boundary of the computational domain. For electrically large objects, a domain decomposition method (DDM) allows a considerable reduction of its numerical complexity by decomposing the initial problem into several coupled subproblems that are solved independently. Such an hybrid DDM has been presented in [1–3] that is based, essentially, on the DDM proposed by Després et al. [4]: the subdomains (SDs) are coupled via a Robin transmission condition (TC) that ensures the uniqueness of the solutions and the convergence of the iterative algorithm. In [1,2], the convergence is accelerated by partitioning the computational domain into concentric SDs (onion-like partition), and Després' DDM algorithm has been modified accordingly. However, for a perfect electric conductor (PEC) coated with materials, it may happen that this algorithm converges rapidly when the first (innermost) interface is located in free-space only. For an electrically large object and high index materials, this entails a prohibitively large number of volume unknowns inside the first SD.

A possible way to overcome this problem is to employ FETI-like methods [5–13]: the interior unknowns in each SD are eliminated by performing a Schur complement, and only the resulting system with the unknowns on the interfaces between the SDs is considered. This allows a reduction of the memory size and the use of a Krylov iterative solver. However, the size of

* Tel.: +33 05 57 04 50 10.
E-mail address: bruno.stupfel@cea.fr

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doi:10.1016/j.jcp.2009.10.015

this system may still be very large, preconditioning is generally necessary, and uniqueness of the solutions is not always guaranteed. Another possibility is to enhance the performances of Després' original TCs. Without pretending to be exhaustive, we may mention numerical TCs [8,18], second order TCs – that involve second order tangential derivatives of the fields on the interfaces – [15,7,16,23], and zero order TCs defined for the two following model problems: unbounded homogeneous medium with a planar interface [17,23], and a 2D coated PEC circular cylinder with one circular interface located in free-space [19], the solution of which is also easily obtained in closed-form. In both cases, Després' algorithm only is considered.

In this paper, we complexify the latter 2D model by adding one or two interfaces inside the coating, in order to approach more closely a real-world problem. Besides, the onion-like (1D) DDM algorithm [15] is employed, that is known to converge more rapidly than the one originally proposed by Després. The model problem with two interfaces is presented in Section 2, together with the definitions of the exact TCs, of the global system reduced to the unknowns on the interface, and of the radius of convergence of the algorithm. The performances of the approximate TCs proposed, e.g., in [17,19] are investigated in Section 3, as well as those of the relaxed algorithm in [15]; also, the issues pertaining to the uniqueness of the solutions and convergence of the algorithm are addressed. More efficient TCs are presented and numerically evaluated in Section 4, and the influence of an additional TC on the convergence is investigated in Section 5. The case of an elliptical PEC cylinder with two free-space subdomains only is investigated in Section 6, and conclusions are proposed in Section 7.

2. Model problem: 2D circular cylinders

We consider the 2D geometry (translationally invariant along z) sketched in Fig. 1. It is embedded in free-space and illuminated by the plane wave $u^{inc} = e^{ik_0x}$ where $k_0 = 2\pi f/c$ is the free-space wave number (c is the light velocity), and the time dependence $\exp(2i\pi ft)$ is assumed and suppressed throughout. The Helmholtz equation $\Delta u + k^2u = 0$ is solved with an exact radiation condition at infinity – for example an IE – and $k = k_0\sqrt{\epsilon\mu}$ where ϵ, μ are the relative permittivity and permeability of the material, being both equal to one in free-space. S_0 is the surface of a PEC:

$$\text{TM}: \; u(S_0) = 0; \quad \text{TE}: \; \partial_n u(S_0) = 0 \tag{1}$$

u is the total field, equal to E_z ($\eta_0 H_z$) in TM (respectively TE) polarization ($\underline{E}$, $\underline{H}$ are the electric and magnetic fields and η_0 is the free-space impedance); ∂_n stands for $\underline{n}.\underline{\nabla}$ where $\underline{n}$ is the outward normal to S_0. This problem is representative of a real-world one: a TC on S1 inside the material and another one on S3 for the hybridization with the IE in free-space [1,2]. An additional interface located inside the material will be considered in Section 5. A general solution of the Helmholtz equation in polar coordinates is

$$u(r,\theta) = \sum_{m=-\infty}^{+\infty} [a_m J_m(kr) + b_m H_m(kr)]e^{im\theta} \tag{2}$$

where $J_m(x), H_m(x) \equiv H_m^{(2)}(x)$ are the Bessel and Hankel functions of order m, and $k = k_0$ for $r \geqslant R_2$. Note that $e^{ik_0x} = \sum_{m=-\infty}^{+\infty} i^m J_m(k_0r)e^{im\theta}$ implies

$$a_{-m} = (-1)^m a_m \quad b_{-m} = (-1)^m b_m \tag{3}$$

2.1. Exact solution

The coefficients for the exact solution that will serve as a reference are:

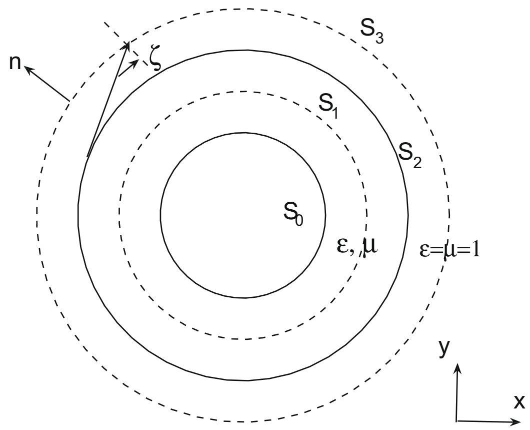

Fig. 1. R_i is the radius of circle S_i, $0 \leqslant i \leqslant 3$. S_0 is a PEC surface; ϵ and μ are constant for $R_0 \leqslant r \leqslant R_2$ and $\epsilon = \mu = 1$ for $r \geqslant R_2$. TCs are prescribed on S_1 and S_3. The normals $\underline{n}$ to the circles are all oriented outward. The arrow represents a ray tangent to S_2 and incident on S_3 with the angle ζ (see Section 4.1.4).