Journal of Computational Physics 237 (2013) 1–20

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Biot-JKD model: Simulation of 1D transient poroelastic waves with fractional derivatives

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ARTICLE INFO

Article history:
Received 29 June 2012
Received in revised form 5 December 2012
Accepted 6 December 2012
Available online 19 December 2012

Keywords:
Porous media
Elastic waves
Biot-JKD model
Fractional derivatives
Time splitting
Finite difference methods
Cartesian grid

ABSTRACT

A time-domain numerical modeling of Biot poroelastic waves is presented. The viscous dissipation occurring in the pores is described using the dynamic permeability model developed by Johnson–Koplik–Dashen (JKD). Some of the coefficients in the Biot-JKD model are proportional to the square root of the frequency: in the time-domain, these coefficients introduce order 1/2 shifted fractional derivatives involving a convolution product. Based on a diffusive representation, the convolution kernel is replaced by a finite number of memory variables that satisfy local-in-time ordinary differential equations. Thanks to the dispersion relation, the coefficients in the diffusive representation are obtained by performing an optimization procedure in the frequency range of interest. A splitting strategy is then applied numerically: the propagative part of Biot-JKD equations is discretized using a fourth-order ADER scheme on a Cartesian grid, whereas the diffusive part is solved exactly. Comparisons with analytical solutions show the efficiency and the accuracy of this approach.

1. Introduction

Porous media consist of a solid matrix within which fluids can circulate freely. The propagation of waves in these media has many crucial implications in applied mechanics, in situations where materials such as industrial foams, spongious bones [34] and petroleum rocks [3] have to be characterized, for example. The poroelastic model originally developed by Biot in 1956 [1] includes two classical waves (one "fast" compressional wave and one shear wave), in addition to a second "slow" compressional wave, which is highly dependent on the saturating fluid. This slow wave was observed experimentally in 1981 [32], thus confirming the validity of Biot's theory.

Two frequency regimes have to be distinguished when dealing with poroelastic waves. One of the main problems is how to model the dissipation of mechanical energy. In the low-frequency range (LF) [1], the viscous boundary layer that develops in the fluid is large in comparison with the diameter of the pores, and the viscous efforts are proportional to the relative velocity of the motion between the fluid and solid components. In the high-frequency range (HF), modeling the dissipation is a more delicate task: Biot first presented an expression for particular pore geometries [2]. In 1987, Johnson–Koplik–Dashen (JKD) [19] published a general expression for the dissipation in the case of random pores. The viscous efforts depend in this model on the square root of the frequency of the perturbation. When writing the evolution equations in the time domain, time fractional derivatives are introduced, which involves convolution products with singular kernels [26]. Analytical solutions have been derived in simple academic geometries and homogeneous media [13].

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http://dx.doi.org/10.1016/j.jcp.2012.12.003

Many numerical methods have been developed in the LF regime: see [5] and the introduction to [7] for general reviews. In the HF regime, the fractional derivatives greatly complicate the numerical modeling of the Biot-JKD equations. The past values of the solution are indeed required in order to evaluate these convolution products, which means that the time evolution of the solution must be stored. This of course greatly increases the memory requirements and makes large-scale simulations impossible. To our knowledge, only two approaches to this problem have been proposed so far in the literature. The first approach consisted in discretizing the convolution products [27], and the second one was based on the use of a diffusive representation of the fractional derivative [25,36]. In the latter approach, the convolution product is replaced by a continuum of diffusive variables – or memory variables – satisfying local differential equations [17]. This continuum is then discretized using appropriate quadrature formulas, resulting in the Biot-DA (diffusive approximation) model.

However, the diffusive approximation proposed in [25] has three major drawbacks. First, the quadrature formulas make the convergence towards the original fractional operator very slow. Secondly, in the case of small frequencies, the Biot-DA model does not converge towards the Biot-LF model. Lastly, the number of memory variables required is not specified. The aim of the present study is therefore to develop a new diffusive approximation method in which these drawbacks do not arise. Since it is proposed here to focus on the discretization of the fractional derivatives, we will deal only with the 1-D equations of evolution in homogeneous media, so that the shear wave will not be considered. However, the strategy proposed here can be extended quite straightforwardly to 2D and 3D geometries, as discussed below.

This paper is organized as follows. The original Biot-JKD model is briefly outlined in Section 2 and the principles underlying the diffusive representation of fractional derivatives are described. The decrease of energy and the dispersion analysis are addressed. In Section 3, the method used to discretize the diffusive model is presented: the diffusive approximation thus obtained is easily treatable by computers. Following a similar approach than in viscoelasticity [15], the coefficients of the model are determined using an optimization procedure in the frequency range of interest, giving an optimum number of additional computational arrays. The numerical modeling is addressed in Section 4, where the equations of evolution are split into two parts: a propagative part, which is discretized using a fourth-order scheme for hyperbolic equations, and a diffusive part, which is solved exactly. Some numerical experiments performed with realistic values of the physical parameters are presented in Section 5. In Section 6, a conclusion is drawn and some futures lines of research are given.

2. Physical modeling

2.1. Biot model

The Biot model describes the propagation of mechanical waves in a macroscopic porous medium consisting of a solid matrix saturated with a fluid circulating freely through the pores [1,3,4]. It is assumed that

- the wavelengths are large in comparison with the diameter of the pores;
- the amplitude of the perturbations is small;
- the elastic and isotropic matrix is completely saturated with a single fluid phase;
- the thermo-mechanical effects are neglected.

This model involves 10 physical parameters: the density ρ_f and the dynamic viscosity η of the fluid; the density ρ_s and the shear modulus μ of the elastic skeleton; the porosity $0 < \phi < 1$, the tortuosity $a \geqslant 1$, the absolute permeability at null frequency κ, the Lamé coefficient λ_f and the two Biot's coefficients β and m of the saturated matrix. The following notations are introduced

$$\begin{aligned} &\rho_w = \frac{a}{\phi}\rho_f, \quad \rho = \phi\rho_f + (1-\phi)\rho_s, \quad \chi = \rho\rho_w - \rho_f^2 > 0, \\ &\lambda_0 = \lambda_f - m\beta^2, \quad C = \lambda_0 + 2\mu > 0. \end{aligned} \tag{1}$$

Taking u_s and u_f to denote the solid and fluid displacements, the unknowns in 1D are the elastic velocity $v_s = \frac{\partial u_s}{\partial t}$, the filtration velocity $w = \frac{\partial \mathcal{W}}{\partial t} = \phi \frac{\partial}{\partial t}(u_f - u_s)$, the elastic stress σ, and the acoustic pressure p. The constitutive laws are

$$\sigma = (\lambda_f + 2\,\mu)\,\varepsilon - m\,\beta\,\xi, \tag{2a}$$

$$p = m(-\beta\,\varepsilon + \xi), \tag{2b}$$

where $\varepsilon = \frac{\partial u_s}{\partial x}$ is the strain and $\xi = -\frac{\partial \mathcal{W}}{\partial x}$ is the rate of fluid change. On the other hand, the conservation of momentum yields

$$\rho\frac{\partial v_s}{\partial t} + \rho_f\frac{\partial w}{\partial t} = \frac{\partial \sigma}{\partial x} + f_b, \tag{3a}$$

$$\rho_s\frac{\partial v_s}{\partial t} + \rho_w\frac{\partial w}{\partial t} + \frac{\eta}{\kappa}F * w = -\frac{\partial p}{\partial x} + f_f, \tag{3b}$$