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A phase field method for simulating morphological evolution of vesicles in electric fields

Ling-Tian Gao [a], Xi-Qiao Feng [a,*], Huajian Gao [b]

[a] Institute of Biomechanics and Medical Engineering, FML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[b] Division of Engineering, Brown University, Providence, RI 02912, USA

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ABSTRACT

A phase field method is developed to investigate the morphological evolution of a vesicle in an electric field, taking into account coupled mechanical and electric effects such as bending, osmotic pressure, surface tension, flexoelectricity, and dielectricity of the membrane. The energy of the system is formulated in terms of a continuous phase field variable and the electric potential. The governing equations of the phase field and the electric field are solved using the Galerkin weighted residual approach. The validation and robustness of the algorithm are verified by direct comparisons of the obtained numerical solutions with relevant experimental results. The morphological evolution of an axisymmetric vesicle under an electric field is studied in detail. The results demonstrate that the present method can simulate complex morphological evolutions of vesicles under coupled mechanical–electrical fields.

1. Introduction

Cell membrane plays a crucial role in many biological processes owing to their unique physical properties [1,2]. Cells can sense their environment and respond to external stimuli [3,4]. Such biological processes as growth, hybridization, migration, proliferation and differentiation of cells are closely related to the electrophysiological properties of cell membrane [3]. Various phenomena including electroporation, electrofusion, electrophoresis, and electro-deformation have been widely utilized in biophysical, biochemical and biomechanical studies of cells such as transfer/delivery of genes, proteins, antibodies or drugs into cells, separation of different kinds of biological macromolecules or cells, and measurement of different physical properties of the cell membrane [1,5–12].

In the past several years, considerable effort has been directed towards understanding and predicting the complicated morphological evolution of vesicles in an electric environment [13–19]. For example, Riske and Dimova [16] predicted a prolate-to-oblate transition in an electric field, depending on the ratio between the conductivity coefficients of the inner and outer electrolytes. Besides the electrical field, various effects from the fluid environment around the vesicle, such as the osmotic pressure, surface tension [1,20,21] and shear flow [22–28], also significantly influence the static and dynamic

* Corresponding author. Tel.: +86 10 6277 2934; fax: +86 10 6278 1824.
E-mail address: fengxq@tsinghua.edu.cn (X.-Q. Feng).

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behaviors of the vesicle. Due to the intrinsic complexity of mechanical–electrical coupling and nonlinear geometric deformation of cells, existing theoretical studies have been largely confined to relatively simple configurations (e.g., infinite planes, spheres and ellipses). Recently, Gao et al. [19] developed a more general liquid crystal model for vesicles subjected to arbitrary electric fields, taking into account such effects as elastic bending, osmotic pressure, surface tension, flexoelectricity, dielectricity, and Maxwell electric pressure of the membrane.

Various numerical methods have been developed to investigate vesicle behaviors under mechanical loading, e.g., finite difference method [20] and finite element method [29–31]. In the presence of an electric field, the behavior of vesicles can become significantly more complicated. Accounting for the Maxwell pressure on the inner and outer sides of the membrane, Hyuga et al. [32,33] developed a semi-analytical perturbation method to calculate the static and dynamic deformation of conductive vesicles. However, this method seems to be limited to problems with weak nonlinearity. A boundary integral approach was adopted by Fan and Fedorov [34] to study the interaction between an AFM tip and a biomembrane in a dilute electrolyte solution. However, this type of methods does not allow for a large topological change and their applications are limited to situations where the governing equations in the bulk phase are linear [35]. The level set method [36,37] and arbitrary Lagrange–Euler method (ALE) [38], which have been widely used in simulations of multiphase flows, dynamic bubbles and drops, can also be used to simulate the morphological evolution of vesicles. However, the level set method often faces difficulties in the renormalization procedure, while the mesh mapping, configuration changing and refinement procedure in the ALE strategy can compromise efficiency and accuracy, especially for strongly nonlinear problems like those studied in the present paper.

The phase field method has provided a powerful tool to solve moving boundary problems involving coupled physical fields (e.g., electric, magnetic, fluidic, and chemical fields). This method is based on a phase field which is assumed to be continuous over boundaries and has the physical meaning of an order parameter. The energy of the system is expressed in terms of the phase field variable, and the morphological evolution can be described without explicitly tracking the boundary. The phase field method has some advantages in dealing with moving boundaries, large deformation, morphological singularity and energy dissipation, and it can be used to simulate complicated microstructure evolution (e.g., solidification, phase transformation, grain growth and domain evolution in thin films) [39]. Recently, Biben and Misbah [40] proposed a phase field model to investigate the tumbling of vesicles under shear flow, and they also extended this approach to three-dimensional vesicle dynamics [35]. Du et al. [41] developed a phase field method to systematically analyze the morphological evolution of vesicles. A similar procedure was developed by Campelo and Hernandez-Machado [42] to simulate the dynamic and stationary shapes of vesicles. Du et al. [41] and Campelo and Hernandez-Machado [42] made use of finite difference and spectrum discretization algorithms, respectively. More recently, Du et al. developed a mixed finite element method to study the equilibrium configuration of vesicle membrane [43] and the dynamics of vesicle membranes in incompressible viscous fluids [44]. They also developed an adaptive FEM approach [45] for handling complex shape and topological changes. However, the electric effects have not been addressed in the previously studies.

In this paper, a phase field model, based on the liquid crystal model of Gao et al. [19], is developed to investigate the morphological evolution of vesicles under arbitrary mechanical and electric fields. We will use the finite element method to discretize the phase field variable, which has certain advantages in treating inhomogeneous material properties as well as complex material interfaces and topology. We will examine in some detail the effects of mechanical–electric coupling, inner and outer electrolytes, flexoelectricity and dielectricity on the morphological evolution of a vesicle and compare the numerical results to relevant experiments.

2. Liquid crystal model of vesicles under mechanical and electric fields

Experimental observations have demonstrated that cell membranes are constructed based on the general principles of liquid crystal lipid bilayers [9,46]. The elastic theory of liquid crystal biomembranes has been successfully applied to study the morphological change, adhesion, and some other related problems of cell membranes [47–50]. However, there is surprisingly little investigation on the effect of flexoelectricity on the behavior of cell membranes. When subjected to strong electric pulses, some unusual deformations of vesicles changing among disc-, square-, and tube-like shapes have been experimentally observed. These phenomena cannot be explained by a liquid crystal model that disregards the electric conductivity of the cell membrane and the electrolyte. In response to recent advances in experimental observations, we proposed a more general electromechanical liquid crystal model of cell membranes based on Eringen's micropolar theory [51]. The model accounts for contributions of elastic bending, osmotic pressure, surface tension, flexoelectric and dielectric effects under various types of mechanical and electrical fields.

In this section, the liquid crystal model of vesicles under mechanical and electric fields [19] is briefly reviewed.

Consider a quasi-static configuration of a dielectric lipid vesicle in an electric field, the Helmholtz free energy of the system can be expressed as

$$F = F_{\mathrm{bm}} + F_{\mathrm{fm}} + F_{\mathrm{dm}} + F_{\mathrm{de}}, \tag{1}$$

where $F_{\mathrm{bm}}, F_{\mathrm{fm}}$, and F_{dm} are the elastic bending energy, the flexoelectric energy, and the dielectric energy of the membrane, respectively, and F_{de} is the dielectric energy of the electrolyte.

According to Helfrich [52], the elastic bending energy is given by