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A fast algorithm for modeling multiple bubbles dynamics

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Abstract

This work presents the development of a numerical strategy that combines the fast Fourier transform on multipoles (FFTM) method and the boundary element method (BEM) to study the physics of multiple bubbles dynamics in moving boundary problems. The recent FFTM method can be employed to speedup the resolution of the boundary integral equation. However, one major drawback of the method is that its efficiency deteriorates quite significantly when the problem is spatially sparsely populated, as in the case where multiple bubbles are well separated. To overcome this deficiency, a new version of FFTM with clustering is proposed (henceforth called FFTM Clustering). The new algorithm first identifies and groups closely positioned bubbles. The original FFTM is then used to compute the potential contributions from the bubbles within its own group, while contributions from the other separated groups are evaluated via the multipole to local expansions translations operations directly. We tested the FFTM Clustering on several multiple bubble examples to demonstrate its effectiveness over the original FFTM method and vast improvement over the standard BEM. The high efficiency of the FFTM Clustering method allows us to simulate much larger multiple bubbles dynamics problems within reasonable time. Some physical behaviors of the multiple bubbles are also presented in this work.

Keywords: Bubble dynamics; Boundary integral method; Multipole approximations; Fast Fourier transform on multipoles

1. Introduction

A better understanding of the physics of multiple bubble dynamics is important for a wide range of applications including underwater explosions, biomedical and chemical processes [1–7]. Numerical modeling and simulation has proven to be an effective approach to study the complex phenomena of multiple bubbles dynamics. Boundary element method (BEM) is one effective numerical tool for solving such dynamical boundary value problem, with previous works by Blake and Gibson [1], Guerri et al. [7], Wang et al. [8,9], Zhang et al. [10], Wang [11], Rungsiyaphornrat et al. [12], Best and Kucera [13] and others.

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BEM has the distinct feature of reducing the problem dimension by 1, that is, only the boundaries of the bubbles and structures need to be discretized, and this greatly simplifies the pre- and post-processing tasks. Unfortunately, conventional BEM generates a dense matrix system, which requires $O(N^2)$ memory storage requirements and operations to solve with iterative methods [14,15]. This poses new challenges, and also provides the motivation to devise more efficient methods, to the simulations of large-scale problems, such as in multiple bubble dynamics, where N can easily exceeds several thousands.

Numerous fast algorithms were developed over the last few decades, including the hierarchical-based tree algorithm [16,17], fast multipole method (FMM) [18–20], and the FFT-based methods such as the particle–particle–particle-mesh (PPPM) methods [21], precorrected-FFT approach [22] and the recent fast Fourier transform on multipoles (FFTM) [23,24]. The efficiency of the former group comes from the use of multipole expansions theories and a highly effective hierarchical structure, where multipole and local expansions are translated efficiently up/down the oct-tree when evaluating the far field potentials. As for the FFT-based methods, the speedup is gained from the use of FFT algorithms [25] for computing the discrete convolutions. Other fast techniques include the sparsification of the dense matrix systems by using singular value decomposition (SVD) [26], wavelet transforms [27] and the multilevel methods for the fast solution of integral equations [28].

In this paper, the fast Fourier transform on multipoles (FFTM) coupled with BEM is implemented to solve the boundary integral equation that governs the dynamics of multiple bubbles. However, its efficiency suffers significantly when the problem is spatially sparse, that is, quite a substantial portion of the problem domain is empty (for example, in the case where the bubbles were placed widely apart). Here, we suggest an improvement with a clustering approach. The new algorithm first identifies and groups closely positioned bubbles, based on their relative distances. Then the elements interactions within the groups itself are evaluated rapidly using FFTM, while interactions among the different groups are evaluated directly via multipole to local translation operations. It is demonstrated that the new approach can performs significantly faster than the original FFTM method without compromising the accuracy.

This paper is organized as follows. Sections 2 and 3 briefly outline the mathematical formulation of the bubble dynamics simulation model, and the mathematical preliminaries of the multipole approximations theories, respectively. In Section 4, the FFTM and the FFTM Clustering algorithms are presented for the bubble dynamics problem. Then in Section 5, we present some numerical studies on the accuracy and effectiveness of the FFTM Clustering method. In Section 6, we study the physics of multiple explosion bubbles. Finally, the major findings of this study are summarized in Section 7.

2. Mathematical formulation of the bubble dynamics model

2.1. Boundary integral equation for potential flow problem

Assume that the fluid domain around the bubbles is inviscid, incompressible and irrotational, the governing equation for such a flow is the Laplace equation, i.e. $\nabla^2\Phi = 0$, where Φ is the velocity potential and the velocity vector is defined as $\boldsymbol{u} = \nabla\Phi$. Using Green's theorem, the problem can be rewritten as the following boundary integral equation:

$$c(\boldsymbol{x})\Phi(\boldsymbol{x}) = \int_\Gamma \left[\frac{\partial\Phi(\boldsymbol{y})}{\partial n}\frac{1}{\|\boldsymbol{x}-\boldsymbol{y}\|} - \Phi(\boldsymbol{y})\frac{(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{n}}{\|\boldsymbol{x}-\boldsymbol{y}\|^3}\right]\mathrm{d}\Gamma(\boldsymbol{y}), \tag{1}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ correspond to the field and source points, respectively, $\boldsymbol{n}$ is the normal vector directed away from the fluid, and $c(\boldsymbol{x})$ is the solid angle defined at the location $\boldsymbol{x}$. Applying the collocation BEM scheme to Eq. (1), with the geometry and the physical variables being approximated by discrete linear triangular elements, leads to a discrete model defined by its nodal variables. Finally, applying the appropriate boundary conditions leads to a dense linear system of equations. As mentioned above, solving the dense matrix system becomes computational intensive for large-scale problems. In Section 4, we present the FFTM algorithms that can solve the problem more efficiently, with minimal loss in accuracy.