Journal of Computational Physics 228 (2009) 3173–3181

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Short Note

Shape and topology optimization for electrothermomechanical microactuators using level set methods

Zhen Luo [a], Liyong Tong [a,*], Haitao Ma [b]

[a] School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia
[b] State Key Laboratory of Subtropical Building Science, Department of Civil Engineering, South China University of Technology, Guangzhou 510641, China

ARTICLE INFO

Article history:
Received 11 July 2008
Received in revised form 1 October 2008
Accepted 5 January 2009
Available online 22 January 2009

Keywords:
Shape and topology optimization
Level set methods
Multiphysics actuators
Compliant mechanisms
Radial basis functions

ABSTRACT

In this short note, a shape and topology optimization method is presented for multiphysics actuators including geometrically nonlinear modeling based on an implicit free boundary parameterization method. A level set model is established to describe structural design boundary by embedding it into the zero level set of a higher-dimensional level set function. The compactly supported radial basis functions (CSRBF) are introduced to parameterize the implicit level set surface with a high level of accuracy and smoothness. The original more difficult shape and topology optimization driven by the Hamilton–Jacobi partial differential equation (PDE) is transferred into a relatively easier parametric (size) optimization, to which many well-founded optimization algorithms can be applied. Thus the structural optimization is transformed to a numerical process that describes the design as a sequence of motions of the design boundaries by updating the expansion coefficients of the size optimization. Two widely studied examples are chosen to demonstrate the effectiveness of the proposed method.

1. Introduction

This short note addresses a parameterization level set method for structural shape and topology optimization of compliant actuators involving multiphysics and geometrical nonlinearity. A variety of methods have been developed for topology optimization such as the homogenization method [5], solid isotropic material with penalization (SIMP) approach [31,6] and evolutionary structural optimization (ESO) [28]. Recently, a new family of methods has emerged for shape and topology optimization problems based on the level set method which was presented by Osher and Sethian [16]. Sethian and Wiegmann [22] are among the first few researchers who introduced the standard level set method [16] into shape and topology optimization on a fixed Eulerian grid. This method is further developed by a number of researchers [17,1,26]. The main feature of these methods is to describe the front implicitly as the zero level set of a higher-dimensional scalar function, and then a velocity field is incorporated to advance the front propagation by directly solving the Hamilton–Jacobi PDE with explicit schemes. However, it should be pointed out that numerical difficulties related to the CFL (Courant–Friedrichs–Lewy) condition, velocity extension scheme and reinitialization procedure limit the application of the level set method to shape and topology optimization problems, because complicated PDE solving procedures are often involved in these numerical procedures [18,22]. In particular, the final design depends on the initial guess because no nucleating mechanism is included to create new holes inside the design domain [10]. To overcome these numerical difficulties, several parameterization or equivalent level set methods have been proposed without solving the Hamilton–Jacobi PDEs via explicit schemes [4,11,13–15].

* Corresponding author. Tel.: +61 2 93516949; fax: +61 2 93514841.
E-mail addresses: zluo@aeromech.usyd.edu.au (Z. Luo), ltong@aeromech.usyd.edu.au (L. Tong), maht@scut.edu.cn (H. Ma).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2009.01.010

The possible applications of topology optimization methods are numerous but an attractive one is the systematic design of actuators in MEMS [2]. As far as the multiphysics actuator is concerned, electrostatics, electromagnetic, electrothermoelastic, piezoelectric and photostrictive principles can be applied as actuation authorities to drive mechanical components to complete specified performance. In this work, the electrothermoelastic action induced by non-uniform Joule heating is considered as the actuation disciplinary. Such multiphysics actuators with embedded actuation have been widely studied recently due to many of its advantages [23,29,30], such as a relatively large displacement with low voltage, good controllability, easy microfabrication and amenability to any practical conducting materials. Multiphysics actuators can be regarded as a sort of electro-thermally actuated compliant mechanisms [12,23] in which fewer parts, fewer assembly processes and no lubrication are required. A promising method for the design of compliant mechanisms is to apply topology optimization methods [7] to generate compliant mechanisms with distributed compliance [13,23,29]. This method is especially suitable for producing compliant micro-devices in MEMS, because the difficulties associated with fabrication and assembly can be avoided.

Luo et al. [13] has presented a parametric level set method for shape and topology optimization problems using CSRBFs [20,27]. This short note naturally extends the parameterization level set method [13] to more advanced shape and topology optimization problems involving multiphysics modeling and geometrically nonlinear analysis.

2. Level set based parameterization scheme

The level set model is mathematically described as a Hamilton–Jacobi equation [16,21]

$$\frac{\partial \Phi(\mathbf{x},t)}{\partial t} - \mathbf{v}_n|\nabla\Phi| = 0, \quad \Phi(\mathbf{x},0) = \Phi_0(\mathbf{x}), \tag{1}$$

where $\Phi(\mathbf{x},t)$ is a Lipschitz continuous scalar function and t is the pseudo-time to enable the dynamic process of the level set surface, and $\mathbf{v}_n$ is the normal velocity field. Hence the movement of the boundary is just a question of transporting the level set function along the normal direction according to a series of solutions from the Hamilton–Jacobi PDE. In general, an analytical level set function is unknown and so in standard level set methods [16,21] an explicit time-marching scheme is indispensable to enable the discrete level set processing.

RBFs have experienced considerable developments due to their favorable interpolation behaviors in multivariate approximations [9]. Centrally positioning the RBF functions at pre-specified knots in the design domain, the original level set function can be described as

$$\Phi(\mathbf{x},t) = \boldsymbol{\varphi}(\mathbf{x})^T\boldsymbol{\alpha}(t) = \sum_{i=1}^{N}\phi_i(\mathbf{x})\alpha_i(t), \tag{2}$$

where the vectors

$$\boldsymbol{\varphi}(\mathbf{x}) = [\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \ldots, \phi_N(\mathbf{x})]^T \in \mathbb{R}^N \tag{3}$$

consists of C^6 continuity shape functions [27] and the expansion coefficients are given by

$$\boldsymbol{\alpha}(t) = [\alpha_1(t), \alpha_2(t), \ldots, \alpha_N(t)]^T \in \mathbb{R}^N. \tag{4}$$

In doing so, the interpolation of the level set function can be uniquely determined in terms of the given data at RBF knots. If these knots are properly distributed in the design domain, they can interpolate the level set function with desired smoothness and completeness, and the continuity of the interpolant can be guaranteed via the continuity of both the shape functions and their partial derivatives. The present interpolation scheme is based on the assumption that all the knots are fixed in the design domain, leading to a separation of the space and time of the original Hamilton–Jacobi PDE (shape functions are spatial while the coefficients are temporal). It is noted that the compact support should be appropriately selected to ensure both the nonsingularity of the interpolation and the computational efficiency [9,27]. This study suggests a support radius that is selected as 2–4 times of the element length according to numerical experience.

The level set model in Eq. (1) is therefore re-written as

$$\boldsymbol{\varphi}(\mathbf{x})^T\dot{\boldsymbol{\alpha}}(t) - \mathbf{v}_n|(\nabla\boldsymbol{\varphi})^T\boldsymbol{\alpha}(t)| = 0. \tag{5}$$

According to Eq. (5), we can get the following normal velocity field:

$$\mathbf{v}_n = \frac{\boldsymbol{\varphi}(\mathbf{x})^T}{|(\nabla\boldsymbol{\varphi})^T\boldsymbol{\alpha}(t)|}\dot{\boldsymbol{\alpha}}(t), \quad \text{where } \dot{\boldsymbol{\alpha}}(t) = \frac{d\boldsymbol{\alpha}(t)}{dt}. \tag{6}$$

It should be noted that Eq. (5) implies a natural extension of $\mathbf{v}_n$ to all knots over the entire design domain rather than only the points on the boundary. However, this study only works out the expression of $\mathbf{v}_n$ but does not need to explicitly calculate it as [1,26].