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Novel and efficient FDTD implementation of higher-order perfectly matched layer based on ADE method

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ABSTRACT

Based on the stretched coordinate perfectly matched layer (SC-PML) formulations and the auxiliary differential equation (ADE) method, an efficient and unsplit-field implementation of the higher-order PML scheme with more than one pole is proposed to truncate the finite-difference time-domain (FDTD) lattices. The higher-order PML has the advantages of both the conventional PML and the complex frequency shifted PML (CFS-PML) in terms of absorbing performances. The proposed algorithm is validated through two numerical tests carried out in three dimensional and two dimensional domains. It is shown in the numerical simulations that the proposed PML formulations with the higher-order scheme are efficient in terms of attenuating both the low-frequency propagating waves and evanescent waves and reducing late-time reflections, and also hold much better absorbing performances than the conventional SC-PML and the convolutional PML (CPML) with the CFS scheme.

1. Introduction

Since the introduction of the perfectly matched layer (PML) absorbing boundary condition by Bérenger [1], various modified PMLs have been presented to terminate the finite-difference time-domain (FDTD) lattices. With the advantage of simple implementation in the corners and the edges of the PML regions, the stretched coordinate PML (SC-PML) [2] was proposed through mapping Maxwell's equations into a complex stretched coordinate space. As original Bérenger's PML, the SC-PML formulations in [2] are ineffective at absorbing the evanescent waves. Recently, the complex frequency shifted PML (CFS-PML) [3], implemented by simply shifting the frequency dependent pole off the real axis and into the negative-imaginary half of the complex plane, has drawn considerable attention due to the fact that this PML is efficient in attenuating the low-frequency evanescent waves and reducing late-time reflections [4]. In [4], the convolutional PML (CPML), based on the SC-PML formulations and the convolution theorem, was presented in detail to efficiently implement the CFS-PML. However, the CFS-PML would have a poor absorption of low-frequency propagating waves as shown in [5–7]. In [7], the performance of regular PML, complex frequency shifted PML, and second-order PML is studied for the numerical stimulation of waveguide problems. The limitation of each PML is clearly demonstrated. The regular PML is the best choice when only propagating waves are present. The CFS-PML is the best choice if strong evanescent waves are present but low-propagating waves are absent. In more general case, where both low-propagating waves and strong evanescent waves are present, the second-order PML is the best choice. It is shown in the numerical stimulations that the second-order PML can attenuate both low-propagating waves and strong evanescent waves. To overcome the limitations of both the conventional PML and the CFS-PML, the higher-order PML was proposed by Correia, which retains the advantages of both the CFS-PML and

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conventional PML in [7]. It has shown that the second-order PML is highly effective in absorbing both evanescent and low-frequency propagating waves in both open-region and periodic problems in [8]. In [8], the concept of a higher-order PML is proposed, and a split-field implementation of higher-order PML is proposed based on SC-PML and ADE method to truncate FDTD lattices. The proposed higher-order PML in [8] has the advantages of both the conventional PML and the complex frequency shifted PML in terms of absorbing performances. However, besides the drawback of more requirements of the memory and the computational time, the higher-order PML implementation proposed in [8] was difficult to be extended to the case with more than two poles because the polynomial expansion was employed.

In this paper, an efficient implementation of the higher-order PML based on unsplit-field SC-PML formulations and the auxiliary differential equation (ADE) method is proposed. For convenience, this PML is referred to here as the LYYADE PML. Consequently, the proposed higher-order PML scheme requires less memory and computational time as compared with that in [8]. Only the second-order case is described in this paper, but this approach is easy to be applied to any number of poles. Due to using unsplit-field, the proposed formulations can save more auxiliary variables than [8]. Besides, due to applying D-B formulations, referring to [12], Consequently, this PML can be applied to truncate arbitrary media, such as lossy, dispersive, anisotropic, inhomogeneous or nonlinear without any modification and all that is needed is to modify (8) and (9) under consideration. The method is available in [12] to obtain E from D using (8) [and H from B using (9)]. It must be noted that if $\varepsilon_r(\omega)$ [or $\mu_r(\omega)$] are not frequency-dependent, E (or H) formulation should be adopted to reduce memory requirement and save computational time. Finally, in [8], the second-order PML based on the SC-PML was implemented by using the split-field PML formulations. However, besides the drawback of more requirements of the memory and the computational time, the higher-order PML implementation proposed in [8] was difficult to be extended to the case with more than two poles because the polynomial expansion was employed.

2. Formulation

In three-dimensional (3-D) SC-PML regions, the normalized frequency-domain modified Maxwell's curl equations can be written as

$$j\omega\varepsilon_r(\omega)\mathrm{E}(\omega) = c_0\nabla_s \times \mathrm{H}(\omega) \tag{1}$$

$$j\omega\mu_r(\omega)\mathrm{H}(\omega) = -c_0\nabla_s \times \mathrm{E}(\omega) \tag{2}$$

where c_0 is the speed of light in free space, $\varepsilon_r(\omega)$ and $\mu_r(\omega)$ are, respectively, the relative permittivity and permeability of the FDTD computational domain and the operator ∇_s is expressed as

$$\nabla_s = \hat{x}S_x^{-1}\partial_x + \hat{y}S_y^{-1}\partial_y + \hat{z}S_z^{-1}\partial_z \tag{3}$$

where ∂_x, ∂_y and ∂_z are the partial derivatives with respect to x, y and z and S_η ($\eta = x, y, z$) are the complex stretched coordinate metrics, which was originally proposed [1] to be

$$S_\eta = 1 + \sigma_\eta/j\omega\varepsilon_0, \quad (\eta = x, y, z) \tag{4}$$

with the CFS scheme and S_η ($\eta = x, y, z$) are defined as

$$S_\eta = \kappa_\eta + \sigma_\eta/(\alpha_\eta + j\omega\varepsilon_0) \tag{5}$$

where α_η and σ_η are assumed to be positive real and κ_η is real and $\geqslant 1$. To make the PML completely independent of the material properties of the FDTD computational domain, both (1) and (2) can be written in terms of the electric flux density D and the magnetic flux density B as

$$j\omega\mathrm{D}(\omega) = c_0\nabla_s \times (\omega) \tag{6}$$

$$j\omega\mathrm{B}(\omega) = -c_0\nabla_s \times (\omega) \tag{7}$$

where D and B are given by

$$\mathrm{D}(\omega) = \varepsilon_r(\omega)\mathrm{E}(\omega) \tag{8}$$

$$\mathrm{B}(\omega) = \mu_r(\omega)\mathrm{H}(\omega) \tag{9}$$

Consequently, this PML can be applied to truncate arbitrary media, such as lossy, dispersive, anisotropic, inhomogeneous or nonlinear without any modification and all that is needed is to modify (8) and (9) under consideration. The method is available in [12] to obtain E from D using (8) [and H from B using (9)]. It must be noted that if $\varepsilon_r(\omega)$ [or $\mu_r(\omega)$] are not frequency-dependent, E (or H) formulation should be adopted to reduce memory requirement and save computational time.The idea of the higher-order PML was proposed in [8] by generalizing this metric for the case where more than one pole was present. For the second-order PML, S_η is defined as

$$S_\eta = S_{1\eta} \cdot S_{2\eta} = (\kappa_{1\eta} + \sigma_{1\eta}/(\alpha_{1\eta} + j\omega\varepsilon_0)) \cdot (\kappa_{2\eta} + \sigma_{2\eta}/(\alpha_{2\eta} + j\omega\varepsilon_0)) \tag{10}$$