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A boundary focused quadrilateral mesh generation algorithm for multi-material structures

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ABSTRACT

This paper describes a new boundary focused mesh generation algorithm. The algorithm produces quadrilateral meshes from images of multi-material structures. The challenge of meshing such images is to create high quality elements which are aligned with complex material boundaries. The approach proposed in this paper uses the following steps: (1) extract boundaries according to pixel colours; (2) smooth the jagged boundaries; (3) generate geodesic isolines aligned with smoothed boundaries; (4) generate a mesh from the boundaries and isolines; (5) optimize the mesh using the "Pisces" pattern (which is introduced in this paper). Application examples are presented to contrast the reliability and effectiveness of the new algorithm with existing approaches.

1. Introduction

With the development of advanced imaging technology such as computed tomography (CT) and magnetic resonance imaging (MRI), digital data are nowadays widely accepted and used in many areas to describe the complicated multi-material structures with multiple colours, such as in mining [1], medicine [2] and material science [3]. Methods [4–9] for meshing images have been studied in both 2D and 3D to convert the imaging data into finite element models for finite element/finite volume simulations [10–13]. However, these mesh methods usually focus on generating meshes inside the objects rather than creating high quality elements aligned with the boundaries. Since finite element analysis of mechanical behaviour is quite sensitive to element quality and shape especially along material boundaries. Poor quality boundary elements could lead to errors and crashes in numerical simulations [14,15]. For instance, boundaries between different materials are major sources of fracture. Hence they require high quality elements and well-aligned features to address such behaviours [16,17]. Therefore, the goal of this paper is to present a boundary focused quadrilateral mesh generation algorithm and evaluate it in meshing complicated multi-material structures based on image data.

Image-based finite element mesh construction is the subject of on-going research. The simplest way to build quadrilateral meshes from images is to use pixel models [18,19]. However, the drawbacks of this method are evident, jagged boundaries lead to poor results, even errors in simulations [8,20]. To remove such jagged features, methods such as marching cubes [21] and level set [4] are sometimes adopted to create smoothed material boundaries for further mesh generation. Based on these boundaries Zhang et al. [5,7] employed quadtree/octree methods to create meshes for multi-material image data in both 2D and 3D. However, in the generated meshes, element quality around boundaries is poor and there are no aligned elements parallel to the boundaries. Zhang et al. [22] proposed a surface smoothing method to improve element quality. But their

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method has no contribution to generating aligned features in meshes. One method for producing both high quality elements and well-aligned features is "paving" proposed by Blacker and Stepheson [23]. "Paving" forms complete rows of quadrilaterals starting from the boundaries and moving into the interior. The main drawback of paving is the difficulty of resolving the intersection of two fronts which are likely to be overlapped with each other. The detection and resolution of the intersection is time-consuming and complicated. To address these problems, Lee et al. [24] extended Q-Morph [25] to generate quadrilaterals for domain with line constraints. The Q-Morph-based methods maintain the desirable features around boundaries. But they require an existing triangular mesh with a sufficient density to capture the model features; otherwise it will have the same problem as the paving method. Park et al. [26] proposed a direct quadrilateral meshing method to generate meshes with random line constraints. But they still encountered the same difficulty of "paving". To overcome this difficulty, Liu et al. [27] proposed a robust quadrilateral mesh generation method. Based on the Catmull-Clark subdivision [28], it could automatically generate meshes with high quality elements comparable to the Catmull and Clark [28] approach. However, elements around boundaries are still in poor quality and without well-aligned features.

In summary, the current challenge of image-based quadrilateral mesh generation is how to robustly generate high-quality elements for complex multi-material structures especially aligned with material boundaries. To address this issue, a novel boundary focused quadrilateral mesh generation method for multi-material structures is proposed which includes image-based boundary extracting, smoothing, quadrilateral mesh generating and irregular nodes removing. The proposed method is outlined in the following three steps:

(1) Boundary smoothing: a phase-based boundary smoothing method is employed to smooth jagged boundaries.
(2) Quadrilateral mesh generation: geodesic isolines are utilized to generate mesh which includes several layers of well-aligned elements parallel to the smoothed boundaries.
(3) Quadrilateral mesh optimization: a general pattern named "Pisces" is proposed to decrease the number of irregular nodes. It covers most of the specified patterns proposed by authors in Liu et al. [27].

This paper is organized as follows: In Section 2, material boundary (i.e. interface between different colours) smoothing as well as quadrilateral mesh generation and optimization are described. Then in Section 3 application examples are illustrated to show the effectiveness of our approach. Finally, conclusions are given in Section 4.

2. Proposed algorithms and implementation

Boundaries distinguishing different colours (i.e. materials) are important features in numerical simulation. To extract these boundaries, samples between different pixels are created and a multiply material marching cubes algorithm [21] is utilized to generate material boundaries through connecting these samples. The proposed meshing approach starts from these extracted material boundaries.

2.1. Boundary smoothing

Due to the nature of images, the extracted boundaries are usually jagged. As these jagged boundaries affect the element quality of generated quadrilateral meshes, a smoothing approach is proposed to remove the jagged features in a phase by phase manner.

Through the linear least squares method, a phase of a boundary line is described by a quadratic function $f(x) = a + bx + cx^2$. Supposing samples S_{i-1}, S_i and S_{i+1} are sequent on this line, the coordinates of node S_{i-1} and S_{i+1} are x_{i-1}, y_{i-1} and x_{i+1}, y_{i+1} respectively. The location of S_i is optimized by the following equations:

$$\begin{cases} f(x) = a + bx + cx^2 \\ g(x) = \frac{y_{i-1}+y_{i+1}}{2} - \frac{x_{i+1}-x_{i-1}}{y_{i-1}+y_{i+1}}\left(x - \frac{x_{i-1}+x_{i+1}}{2}\right) \\ x_{i-1} < x < x_{i+1} \end{cases} \quad (1)$$

where $g(x)$ is the perpendicular bisector of the segment$S_{i-1}S_{i+1}$.

To demonstrate the proposed phase-based boundary smoothing method, a jagged boundary (Fig. 1(a)) is utilized. Before describing this method, thresholds are introduced to control the process of finding phases: the maximum number of samples on the phase N and the maximum angle between the first and last edge vectors of the phase Ang. We take $N = 6$ and $Ang = \pi/2$ in this paper.

As shown in Fig. 1(b), and the previous smoothed boundary phase consists of samples from a to b. To find a new phase, the sample (i.e. sample c) between a and b is used as the start. And then the new phase is propagated sample by sample until there are N samples or the angle between the first and last edge vectors is larger than Ang. In Fig. 1(b), $\vec{u}$ is the first edge vector and $\vec{v}$ is the last one. Although the angle between $\vec{u}$ and $\vec{v}$ is less than Ang, the propagating process is stopped as d is the sixth sample. The current phase consists of samples from c to d. Eq. (1) is employed to smooth this phase as shown in Fig. 1(c). The final smoothed boundary is illustrated in Fig. 1(d).

The phase-based boundary smoothing method is described as below.