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A grid based particle method for moving interface problems

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ABSTRACT

We propose a novel algorithm for modeling interface motions. The interface is represented and is tracked using quasi-uniform meshless particles. These particles are sampled according to an underlying grid such that each particle is associated to a grid point which is in the neighborhood of the interface. The underlying grid provides an Eulerian reference and local sampling rate for particles on the interface. It also renders neighborhood information among the meshless particles for local reconstruction of the interface. The resulting algorithm, which is based on Lagrangian tracking using meshless particles with Eulerian reference grid, can naturally handle/control topological changes. Moreover, adaptive sampling of the interface can be achieved easily through local grid refinement with simple quad/oct-tree data structure. Extensive numerical examples are presented to demonstrate the capability of our new algorithm.

1. Introduction

The modeling of interfacial motion is an important task in many problems in science and engineering. Numerically, there are two main challenges: (1) representing and tracking a manifold with complicated geometry and dynamics that can develop large deformation, singularities, and even topological changes, (2) coupling between the moving interface and the global dynamics. In this paper we will focus on the first issue and develop a new grid based particle method for representing and tracking a moving interface. The second issue will be reported in our later work.

In general we can classify existing numerical methods for moving interface problems into two categories. The first category is Lagrangian tracking methods, in which the interface is explicitly represented by Lagrangian particles (markers) and its dynamics is tracked by the motion of these particles. Usually these markers are ordered or are connected through either some parametrization or meshes of the interface. For examples, boundary integral methods [13,19], boundary element methods, front tracking method [30,9,28], and etc. belong to this type. Main advantages of the tracking methods are (1) efficient and accurate representation of the interface, (2) simplicity in tracking the motion of particles. However, it is difficult (especially in high dimensions) to maintain a smooth global parametrization or a quasi-uniform mesh for a moving interface with complicated geometry and dynamics involving large deformations or topological changes. The resulting algorithm usually requires reparametrization/remeshing constantly during the evolution as well as complicated reconnection through surgery when topology changes.

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The second category is capturing methods, in which the interface is implicitly embedded in a scalar field function defined on a fixed mesh, such as a Cartesian grid. The interface dynamics is captured by the evolution of the scalar function in an Eulerian framework. For examples, level set methods [18], phase field method [4,1], volume of fluid method [11], and etc. belong to this type. Main advantages of capturing methods are (1) a geometric problem is turned into a partial differential equation (PDE) problem (no parametrization or surface mesh is needed) on a fixed grid with simple data structure, (2) topological changes, such as merging or splitting, can be handled easily in the viscosity sense. However, there are also a few disadvantages for these Eulerian approaches. For examples, due to the implicit representation, capturing methods are usually less accurate and less efficient than tracking method in terms of both interface representation and evolution. Excessive numerical dissipation might be introduced in the computations. Extra effort is needed to determine the interface explicitly. Also solving PDEs based on Cartesian grid makes grid refinement and adaptivity complicated.

Another difficulty for most of the above numerical methods is how to control topological change according to the physics. For example, when two wave fronts meet, they cross each other without interfering. On the other hand, when two burning fronts meet or when two bubbles collide, they merge. Lagrangian tracking methods can easily model the motion of interface passing through each other since particles on the interface are connected locally through parametrization or surface meshes. When two different parts of the interface get close in physical space, they do not feel each other since they are far apart in the parameter space or in the surface meshes. These two segments move independently and pass through each other. However, merging or splitting is a major difficulty for the Lagrangian tracking method as explained above. On the contrary, Eulerian capturing methods embeds the interface in a single valued scalar function. So topological changes such as merging or splitting can be handled easily while interface crossing is difficult to deal with for most capturing methods.

In this paper we propose a new framework to model interface motions which naturally combines and takes advantages of the Lagrangian and the Eulerian formulations. The basic idea is to represent and track the interface explicitly as in the usual Lagrangian methods using quasi-uniform meshless particles, while an underlying Eulerian grid serves as a reference for those particles. Here are a few key features for our method.

1. The interface is represented by particles without mesh or parametrization. This feature allows one to easily add or delete particles, which is important for maintaining a consistent resolution of the interface as well as dealing with topological changes.
2. The sampling of the particles has a one-to-one reference to the underlying grid points which are in the neighborhood of the interface. This Eulerian reference provides both a quasi-uniform sampling of the interface and neighborhood information among meshless particles. These information is useful for local construction of the interface and collision detection of different parts of the interface. The reference is updated with the evolution with no PDE involved on the underlying grid.
3. Adaptive sampling of the interface can be achieved easily through local grid refinement of the underlying grid since no PDE is solved on the grid.
4. Topological change can be controlled using both Lagrangian and Eulerian information available.

In the past, various authors have proposed several ideas to combine Lagrangian and Eulerian approaches for moving interface problems.

The level contour reconstruction methods [23,22] also represented the moving interface using meshless particles. To resample the interface after motion however, the authors proposed first constructing an indicator function (a heaviside function) defined on an underlying Eulerian mesh by solving a Poisson equation, and then finding a particular contour line/surface (not necessary to be 0.5 in order to have conservation of mass) of this indicator function using linear interpolation. This results in a low order reconstruction of the interface. Moreover, since this method was derived to model fluid problems, it is suitable only for problems with interface merging and splitting. One cannot generalize these methods for computing multi-valued solutions.

A front tracking method with an underlying grid was proposed in [8]. The interface is represented by a triangulated mesh. The underlying grid is used to maintain a quasi-uniform triangulation of the interface during the evolution and to reconnect the interface when topological changes take place.

Particle level set method [6] and some related methods [10] were developed based on the level set method but with auxiliary particles. The interface motion is captured by the level set function (by solving an appropriate PDE) as well as is tracked by the particle. Because the particles have better resolution of the interface, they are used to reconstruct/modify the level set function after each time step. The interface is still implicitly represented by the underlying level set function. All these methods are different from ours in which we use meshless particles to represent and track the interface, and we do not solve any PDE on the underlying grid.

Another approach trying to combine a fixed underlying grid with Lagrangian particles is the dynamic surface extension method [25]. In [21], the authors proposed coupling with the closest point method to compute the multi-valued solution of geometrical optics solution in high frequency regime. The method shares some similarity with ours in which grid points from the underlying mesh will be associated to their corresponding closest point on the interface (or foot-point in our method). The key point in these methods is how to update the closest point relation with the dynamics of the interface in an appropriate way such that they can deal with self-intersecting of wave front. Moreover, the interface is never explicitly reconstructed locally. Instead, to restore the property of the closest point, the authors applied simply interpolation techniques which provided approximations to only the new closest points and also normals at these corresponding locations. Therefore,