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Geophysical–astrophysical spectral-element adaptive refinement (GASpAR): Object-oriented *h*-adaptive fluid dynamics simulation

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Abstract

An object-oriented geophysical and astrophysical spectral-element adaptive refinement (GASpAR) code is introduced. Like most spectral-element codes, GASpAR combines finite-element efficiency with spectral-method accuracy. It is also designed to be flexible enough for a range of geophysics and astrophysics applications where turbulence or other complex multiscale problems arise. The formalism accommodates both conforming and non-conforming elements. Several aspects of this code derive from existing methods, but here are synthesized into a new formulation of dynamic adaptive refinement (DARe) of non-conforming *h*-type. As a demonstration of the code, several new 2D test cases are introduced that have time-dependent analytic solutions and exhibit localized flow features, including the 2D Burgers equation with straight, curved-radial and oblique-colliding fronts. These are proposed as standard test problems for comparable DARe codes. Quantitative errors are reported for 2D spatial and temporal convergence of DARe.

Keywords: Spectral element; Numerical simulation; Adaptive mesh; AMR

1. Introduction: a need for high-accuracy dynamic adaptivity

Accurate and efficient simulation of strongly turbulent flows is a prevalent challenge in many atmospheric, oceanic, and astrophysical applications. New simulation codes are needed to investigate such flows in the parameter regimes that interest the geophysics communities. Turbulent flows are linked to many issues in the geosciences, for example, in meteorology, oceanography, climatology, ecology, solar-terrestrial interactions, and solar fusion, as well as dynamo effects, specifically, magnetic-field generation in cosmic bodies by

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turbulent motions. Nonlinearities prevail when the Reynolds number Re is large. The number of 3-dimensional degrees of freedom (d.o.f.) increases as $Re^{9/4}$ as $Re \to \infty$ in the Kolmogorov 1941 framework [16, Section 7.4]. For aeronautic flows often $Re > 10^6$, but for geophysical flows often $Re \gg 10^8$ [11,28]. Also, computations of turbulent flows must contain enough scales to encompass the energy-containing and dissipative scale ranges *distinctly*. Uniform-grid convergence studies on 3D compressible-flow simulations show that in order to achieve the desired scale content, uniform grids must contain at least 2048^3 cells [33]. Today such computations can barely be accomplished. A pseudo-spectral Navier–Stokes code on a grid of 4096^3 uniformly spaced points has been run on the Earth Simulator [19], but the Taylor Reynolds number ($\propto \sqrt{Re}$) is still no more than ≈700, very far from what is required for most geophysical flows. The *main goal of the present code development* is to ask, if the significant structures of the flow are indeed sparse, so that their dynamics can be followed accurately even if they are embedded in random noise, then does dynamic adaptivity offer a means for achieving otherwise unattainable large Re values. Thus, we have developed a dynamic geophysical and astrophysical spectral-element adaptive refinement (GASpAR) code for simulating and studying turbulent phenomena.

Several properties of spectral-element methods (SEMs, [9,29]) make them desirable for direct numerical simulation of geophysical turbulence. Perhaps most significant is the fact that SEMs performed at high polynomial degree are inherently minimally diffusive and dispersive. This property is clearly important when trying to simulate high-Re flows with multiple spatial and temporal scales that characterize turbulence. Also, because SEMs use finite elements, they can be used in very efficient high-resolution turbulence studies in domains with complicated boundaries. It is an important feature that SEMs are naturally parallelizable (e.g., [15]). Equally important, SEMs not only provide spectral convergence when the solution is smooth (see Appendix Eq. (A.3)), but are also effective when the solution is not smooth.

Our goal in this paper is to describe GASpAR and, in particular, the procedures used in our dynamic adaptive refinement (DARe) technique. We provide SEM and DARe algorithm *details* here that are not available elsewhere, in the hope of supporting readers who wish to create their own codes. Furthermore, we propose several linear and nonlinear problems as standards to test fundamental aspects of flows that are encountered in turbulence studies, and use these to test our DARe algorithms. Because these problems have known exact time-dependent solutions, quantitative errors can be reported for DARe simulations. Our code is object-oriented, and we will describe how object-oriented programming serves our purposes. The code is parallelized, but we will discuss this aspect only when it is intrinsic to the algorithms. While we are motivated by the performance potential of SEMs generally [8,34], we do not emphasize performance metrics in the present paper, in favor of focusing on algorithmic detail and solution accuracy.

First we describe (Section 2.2) SEM discretization on a particular class of problems and introduce many of the required formulas, operators, and so forth. We explain (Section 2.4) how continuity is maintained between non-conforming elements. We provide linear-solver details in Section 2.5, and introduce innovations required to solve on non-conforming elements. In Section 2.6, we present our new adaptive-mesh algorithms: how neighboring elements are found, how conformity is established, and the procedures for refinement and coarsening. In Section 2.6.3, we describe a new implementation of element-boundary communication. DARe criteria are discussed in Section 2.6.4. Then, in Section 3 we propose and perform examples from two test-problem classes with time-dependent analytic solutions: the linear advection–diffusion equation (Section 3.2), demonstrating feature tracking of smooth and isolated features; and the 2D Burgers equation (Section 3.3), testing the ability of DARe to track well-defined increasingly sharp structures arising from nonlinear dynamics. In Section 4, we offer some conclusions, as well as comments on potential application of GASpAR to geophysical turbulence simulations.

2. Temporal and dynamically adaptive spatial discretizations

2.1. Adaptive-mesh geometry

Conforming adaptive methods (where entire element boundaries geometrically coincide, as in Fig. 1a) on quadrilaterals and hexahedra are gradually being replaced by non-conforming adaptive methods. One reason is that locally adaptive mesh generation for conforming methods is complicated [30]. Another reason is that