Journal of Computational Physics 228 (2009) 2587–2603

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Simulating the dynamics of flexible bodies and vortex sheets

Silas Alben

School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332-0160, United States

ARTICLE INFO

Article history:
Received 24 February 2008
Received in revised form 2 December 2008
Accepted 15 December 2008
Available online 29 December 2008

PACS:
47.11.–j
47.11.Kb
47.32.C–
47.15.ki
46.40.Jj

Keywords:
Implicit
Coupled
Flow-body
Flexible
Vortex sheet

ABSTRACT

We present a numerical method for the dynamics of a flexible body in an inviscid flow with a free vortex sheet. The formulation is implicit with respect to body variables and explicit with respect to the free vortex sheet. We apply the method to a flexible foil driven periodically in a steady stream. We give numerical evidence that the method is stable and accurate for a relatively small computational cost. A continuous form of the vortex sheet regularization permits continuity of the flow across the body's trailing edge. Nonlinear behavior arises gradually with respect to driving amplitude, and is attributed to the rolling-up of the vortex sheet. Flow quantities move across the body in traveling waves, and show large gradients at the body edges. We find that in the small-amplitude regime, the phase difference between heaving and pitching which maximizes trailing edge deflection also maximizes power output; the phase difference which minimizes trailing edge deflection maximizes efficiency.

© 2008 Elsevier Inc. All rights reserved.

1. Introduction

Computing the large-amplitude motions of flexible bodies in fluids is challenging when the Reynolds number (or inverse viscosity) is large. Resolving the flow accurately requires solving for the position and motion of thin layers of vorticity, which can display highly complex dynamics in even the simplest situations. A fundamental source of complexity is the Kelvin–Helmholtz instability, which leads initially flat layers of vorticity to roll up into spirals [1]. The instability growth rate is maximal at large wave numbers, leading to fast growth in the spatial complexity of thin vortex layers [2].

When the motion of the solid boundary is coupled to the fluid dynamics, an additional challenge arises. The boundary conditions for the fluid solver are now to be imposed on a boundary with location unknown *a priori*. The motion of the body and the flow can only be determined together as a coupled system.

The immersed boundary method has been applied to a wide range of problems in this class [3]. The method's generality makes it suitable for a number of problems. The method uses an Eulerian grid in the fluid, a Lagrangian grid on the body, and an interpolation scheme to communicate forces between the two. At large Reynolds numbers, very fine grids are required to resolve vorticity. Furthermore, in standard formulations convergence is formally second-order in space, though practically first-order convergence often occurs [4,5].

E-mail address: alben@math.gatech.edu

0021-9991/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcp.2008.12.020

A different fluid model considers the infinite Reynolds number limit, in which case the flow is inviscid and irrotational, apart from the vortex layers, which tend to sheets of infinitesimal thickness. Computing the dynamics of isolated vortex sheets in free space began in the 1930s [6], but some of the most fundamental issues in such calculations have been resolved only in the last twenty years [2,7]. These calculations and earlier efforts [8,9] showed that the Birkhoff–Rott equation for the dynamics of a vortex sheet is ill-posed, giving rise to a singularity in the sheet curvature at finite time. Consequently many workers have studied regularized versions of the Birkhoff–Rott equation to obtain smooth problems [10]. Such regularizations have also been used to suppress numerical instabilities [2].

Despite the challenges inherent in the computation of vortex sheets, they are efficient representations of thin shear layers in high Reynolds number flows. They are surfaces in the flow, and hence reduce the dimensionality of the problem by one relative to bulk fluid solvers, which must distribute many grid points across the shear layer.

How vortex sheets are produced at solid surfaces is another challenge which has been addressed recently [11,12], by reformulating the Kutta condition, well-known in classical airfoil theory [13]. These models apply to flows past sharp-edged bodies, which fixes the edge as the location at which the vortex sheet separates from the body. The rate of vorticity flux from the body edge into the sheet is set to make finite the flow velocity at the body edge. Mathematically, this condition removes the singularity which arises generically in potential flow past a sharp-edged body.

Very recently, workers have begun to apply such models to the motions of deforming bodies with prescribed motions [14]. When the body is a flat plate, many of the equations can be formulated analytically. For deforming bodies in prescribed motion, a more general formulation of the equations coupling the body to the flow is required. An additional level of complexity arises when the motion of the deformable body is not prescribed in advance, but is instead coupled to the flow. This is the topic we address here. Because the motion of the body and the strength of vorticity it sheds into the flow are coupled, each can reinforce the other to create a numerical instability unless a special stabilizing approach is found. Here we describe a stable method for such problems, which was recently used to study the large-amplitude dynamics of the flapping-flag instability [15]. The present work examines the numerical method, which was not described in [15]. The present work also gives results in the context of a different problem of scientific interest – the production of vorticity by a passive flexible fin, immersed in an oncoming flow, and driven at the leading edge by a pitching and heaving motion. The linearized version of this problem was studied theoretically in [16], which described some of the basic physics of the generation of thrust forces. Principal among the results was the appearance of an optimal flexibility for thrust, which occurs at the first of a series of resonant-like peaks, each corresponding to an additional half-wavelength of deformation on the fin. In the linearized model, the vortex sheet is a semi-infinite line extending downstream of the fin, and the strength of vorticity on the sheet is simply a travelling wave.

In the general (large-amplitude) version of the problem considered here, the dynamics of the vortex sheet show instead the rolling-up behavior due to the Kelvin–Helmholtz instability. As in the flapping-flag case, the computation is made somewhat easier by the presence of a background flow. The background flow is by no means necessary, but allows for less expensive long-time simulations by moving the vortex sheet steadily away from the body, where the sheet can be approximated by point vortices.

The goal of this work is to describe and present results for a stable and efficient method for coupled vortex sheet-flexible body dynamics, in a fundamental biolocomotion problem. The paper is organized as follows. In Section 2, we present the complete equations for the coupled initial-boundary-value problem consisting of a 2D inviscid flow past a 1D elastic body with a vortex sheet produced at the body's trailing edge. In Section 3 we present the numerical method for this problem, which combines an implicit formulation on the body with an explicit formulation on the free sheet. We also present results on the behavior of the scheme with respect to numerical parameters. In Section 4, we present results with respect to physical parameters. Section 5 summarizes the main results.

2. Flexible body vortex sheet model

We model the tail fin of a swimming fish as a slender elastic filament in a two-dimensional inviscid flow (see Fig. 1). The model fin is an inextensible elastic sheet of length $2L$, mass per unit length ρ_s, and uniform rigidity B, moving under the pressure forces of a surrounding inviscid and incompressible fluid of density (mass per unit area) ρ_f. The fin position is $\zeta(s,t)$, where s is arclength; $-L \leqslant s \leqslant L$. The fin position evolves according to Newton's 2nd law as a geometrically-nonlinear elastica with inertia [17]:

$$\rho_s \partial_{tt}\zeta(s,t) = \partial_s(T(s,t)\hat{s}) - B\partial_s(\partial_s\kappa(s,t)\hat{n}) - [p](s,t)\hat{n}. \tag{1}$$

Here $T(s,t)$ is a tension force which maintains inextensibility, $\kappa(s,t)$ is the fin curvature, and $[p](s,t)$ is the pressure jump across the fin. We have assumed for simplicity that the rigidity B is uniform, and defer a consideration of the spatial distribution of B to future work.

For simplicity we shall represent 2D quantities as complex numbers, so that $\zeta(s,t) = x(s,t) + iy(s,t)$ is the fin position. Here $\hat{s}$ and $\hat{n}$ are complex numbers representing the unit tangent and normal vectors to the fin, respectively. Therefore we have $\hat{s} = \partial_s\zeta = e^{i\theta(s,t)}$, where $\theta(s,t)$ is the local tangent angle, and $\hat{n} = ie^{i\theta(s,t)}$. We shall make extensive use of the following identity between the scalar product of two real vectors (a,b) and (c,d) and the product of the complex numbers $w_1 = a + ib$ and $w_2 = c + id : (a,b)\cdot(c,d) = \mathrm{Re}(w_1\bar{w}_2)$, where the bar denotes the complex conjugate.

Download English Version:

https://daneshyari.com/en/article/521785

Download Persian Version:

https://daneshyari.com/article/521785

Daneshyari.com