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Application of the lattice Boltzmann method to two-phase Rayleigh–Benard convection with a deformable interface

Qingming Chang, J. Iwan D. Alexander *

Department of Mechanical and Aerospace Engineering, National Center for Space Exploration Research, Case Western Reserve University, Glennan Building, Room 408, 10900 Euclid Avenue, Cleveland, OH 44106, USA

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Abstract

The lattice Boltzmann method (LBM) is extended to include the effects of interfacial tension and its dependence on temperature and is applied to the problem of buoyancy-driven flow in a non-isothermal two-phase system. No a priori assumptions are made regarding the shape and dynamic roles of the interface. The behavior of interface is obtained as part of the solution of the lattice Boltzmann equations. A parametric study of the effects of thermally induced density change, buoyancy, surface tension variation with temperature on interface dynamics, flow regimes and heat transfer is presented.

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Keywords: The lattice Boltzmann method; Multiphase fluid flow; Rayleigh–Benard convection; Double distribution functions; Interface deformation; Boussinesq approximation; Marangoni force

1. Introduction

The Rayleigh–Benard system, in which a fluid is confined between two horizontal parallel plates and maintained at different temperatures, is one of the most studied non-equilibrium hydrodynamic systems.

Most prior work has focused on the case of a single fluid that fills the entire space between the plates [1–4]. However, there are systems of interest in which the fluid consists of two or more layers of immiscible liquids. Each fluid layer shares a common boundary with another fluid layer. Some previous numerical

* Corresponding author. Tel.: +1 216 368 0530; fax: +1 216 368 6445.
E-mail address: iwan@mae.cwru.edu (J.I.D. Alexander).

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studies [5–7] have been applied to the two-fluid Rayleigh–Benard problem but the shape of the interface was assumed to be rigid, flat and horizontal. Zhang and Alexander [8] have addressed the simpler computational problem of flows with curved deformable surfaces in liquid-bridge related problems. Cliffe and Tavener [9] employed an orthogonal mapping technique to solve the location of the deformable interface. However, to avoid the complex issues surrounding moving contact lines and time-dependent interfaces, they limited their studies to steady solutions.

Over the last decade, the lattice Boltzmann method (LBM) has become an established numerical approach in computer fluid dynamics (CFD), because of its capability to simulate flow in multiphase fluids. The underlying concept of the LBM is to incorporate the essential physics of the problem into a simplified kinetic equation such that the correct macroscopic behavior of the fluid is recovered. In contrast to traditional CFD methods which solve macroscopic equations, the LBM simulates fluid flow based on microscopic model or mesoscopic kinetic equations. This intrinsic feature is attractive to those who wish to incorporate microscopic or mesoscopic features and processes that are either not used in, or are difficult to incorporate, in traditional CFD simulation models. In particular, phase segregation and interfacial dynamics, which are essential in multiphase fluids are difficult (but not impossible) to simulate by traditional approaches, can be modeled in LBM by incorporation of molecular interactions. The sharp interface between different immiscible phases can be automatically maintained without any artificial treatment [10]. This feature is especially attractive when there is a need for modeling flow in so-called micro- or even nano-fluidic devices.

For simulating two-phase fluid flows, four LB methods have been used to date: the chromodynamic model proposed by Gunstensen et al. [11] and Grunau et al. [12], the pseudopotential model [13,14], the free-energy model proposed by Swift et al. [15,16] and the index fluid method developed by He [17]. All have their origins in kinetic theory. In some situations, the chromodynamic, pseudopotential and the free-energy model can lead to unphysical behavior, such as the spurious current around interfaces [18], thermodynamic inconsistencies [15] and lack of Galilean invariance [17,19]. The method recently developed by He and Chen [17] employs an index function to track different phases and the interface between them using a mean-field approximation for intermolecular attraction and Enskog's exclusion-volume effect for short range strong repulsion. When the molecular attraction is strong enough, the index function automatically separates into two different phases.

In most multiphase LBE models to date, only mass and momentum conservation is implemented. The macroscopic equations of these models correspond to the Navier–Stokes equation with an equation of state and a constant temperature. However, it is important and sometimes critical to have the capability of simulating thermal effects simultaneously with the fluid flow. Since the LBE method has several attractive features that make it a strong candidate for the simulation of complex fluids with multiple phases and phase transitions, it is necessary to develop the capability of simultaneously solving the energy, momentum and mass balance equations using LBM.

Unfortunately, the simulation of thermal multiphase systems by the LBE method has not yet achieved the same success as that of isothermal flows. Theoretically, a LBE model with energy conservation can be constructed [20–22] to yield a temperature evolution equation at the macroscopic level. However, when inter-particle forces are included, as in the multiphase models, energy conservation is further complicated by the contribution to the internal energy due to interactions between components. For this reason, constructing a non-ideal-gas LBE model with energy conservation is a challenge.

Rather than use a scheme for the energy balance that is based entirely on the LBM, we avoid the difficulties inherent in LBM and use a hybrid model that involves the LBM for the momentum and mass conservation equations and continuum model for the energy conservation. In other words, a two-phase LBM based on the double population model is combined with a scalar energy equation. Such an approach has been used by Filipova and Hanel [23] for instant to simulate combustion. Using the hybrid lattice Boltzmann finite-difference simulation, two-fluid Rayleigh–Benard convection is investigated. No a priori assumptions about the shape and dynamical behavior of the interface are made.