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The Riemann problem and a high-resolution Godunov method for a model of compressible two-phase flow

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Abstract

This paper considers the Riemann problem and an associated Godunov method for a model of compressible two-phase flow. The model is a reduced form of the well-known Baer–Nunziato model that describes the behavior of granular explosives. In the analysis presented here, we omit source terms representing the exchange of mass, momentum and energy between the phases due to compaction, drag, heat transfer and chemical reaction, but retain the non-conservative nozzling terms that appear naturally in the model. For the Riemann problem the effect of the nozzling terms is confined to the contact discontinuity of the solid phase. Treating the solid contact as a layer of vanishingly small thickness within which the solution is smooth yields jump conditions that connect the states across the contact, as well as a prescription that allows the contribution of the nozzling terms to be computed unambiguously. An iterative method of solution is described for the Riemann problem, that determines the wave structure and the intermediate states of the flow, for given left and right states. A Godunov method based on the solution of the Riemann problem is constructed. It includes non-conservative flux contributions derived from an integral of the nozzling terms over a grid cell. The Godunov method is extended to second-order accuracy using a method of slope limiting, and an adaptive Riemann solver is described and used for computational efficiency. Numerical results are presented, demonstrating the accuracy of the numerical method and in particular, the accurate numerical description of the flow in the vicinity of a solid contact where phases couple and nozzling terms are important. The numerical method is compared with other methods available in the literature and found to give more accurate results for the problems considered.

Keywords: Multiphase flow; Hyperbolic PDEs; Riemann problems; Godunov methods; Shock waves

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1. Introduction

This paper considers the Riemann problem and an associated high-resolution Godunov method for a system of nonlinear, hyperbolic partial differential equations modeling compressible, two-phase flow. While models of this kind arise in a number of applications, the context of deflagration-to-detonation transition in high-energy condensed-phase explosives provides the motivation for the present effort. A two-phase continuum description of granular explosives has been provided by Baer and Nunziato [1]; also see the contemporaneous study of Butler and Krier [2], the earlier work of Gokhale and Krier [3], and the later papers of Powers et al. [4,5]. The model treats the explosive as a mixture of two phases, the unreacted granular solid and the gaseous product of combustion. Each phase is assigned a set of state variables such as density, velocity, pressure, etc., which are assumed to satisfy balance laws of mass, momentum and energy. A compaction equation and a saturation constraint for the volume fractions of the phases complete the system of equations. The balance laws for each phase are similar to those for an isolated gas, i.e., the Euler equations, except for two important differences. First, the exchange of mass, momentum and energy between the phases appears as source terms in the balance equations. Second, the governing equations, although hyperbolic, are incapable of being cast in a conservative form. Non-conservative terms (also called *nozzling terms* by analogy with similar terms arising in equations that govern flow within a variable-area duct) appear in the equations, and their treatment requires special consideration.

The aim of this paper is twofold. First, we consider the Riemann problem for the homogeneous portion of the governing equations (i.e., with the source terms omitted), and describe an iterative procedure that produces exact solutions for general left and right states of the initial flow. In the Riemann problem the effect of the nozzling terms is confined to the contact discontinuity of the solid phase, across which the volume fraction of each phase changes discontinuously. It is assumed that the discontinuity can be replaced by a layer of finite but vanishingly small thickness within which the volume fractions vary smoothly and the phases interact. This regularization was first proposed in the context of permeation through a porous solid by Asay et al. [6]. An analysis of the layer yields jump conditions for the states of the flow across the solid contact, and allows the contribution of the nozzling terms to be computed in a straightforward and unambiguous fashion. Away from the solid contact the volume fractions are constant so that balance equations for the phases decouple and reduce to Euler equations for the individual phases. In these regions the usual jump conditions across shocks, rarefactions and the gas contact discontinuity apply, and may be used together with the conditions across the solid contact where the phases are coupled to construct an exact solution of the Riemann problem for the mixture.

Next, the solution of the Riemann problem is employed in the development of a high-resolution Godunov-type method [7]. In addition to providing a means to compute a numerical flux at the boundary between neighboring grid cells, the solution of the Riemann problem provides a natural approach to the numerical treatment of the non-conservative terms. The governing equations are integrated over a grid cell. The numerical flux at the boundary emerges from this integral in the standard way following the usual Godunov flux construction. In the case of the non-conservative terms, the integral reduces to a contribution about the solid contact in the solution of the Riemann problem from each cell boundary. With the thin layer structure of the contact discontinuity at hand, this contribution can be computed unambiguously. Thus the resulting numerical method incorporates both the wave structure at cell boundaries and the appropriate behavior of the solution near the solid contact.

A high-resolution method is obtained using a second-order, slope-limited extension of the Godunov method. The approach follows the usual description (see [8,9], for example) except for the treatment of the non-conservative terms which is new. Essentially, the extension for the non-conservative terms involves two parts, one coming from a contribution to the integral of the nozzling terms about the solid contact and the other coming from the integral away from the jump at the solid contact, which arises from the slope correction of the left and right states of the Riemann problems. An improvement in efficiency in the