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A weighted essentially non-oscillatory numerical scheme for a multi-class traffic flow model on an inhomogeneous highway

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Abstract

As a new attempt to solve hyperbolic conservation laws with spatially varying fluxes, the weighted essentially non-oscillatory (WENO) method is applied to solve a multi-class traffic flow model for an inhomogeneous highway. The numerical scheme as well as an analytical study is based upon a modified equivalent system that is written in a "standard" hyperbolic conservation form. Numerical examples, which include the difficult traffic signal control problem, are used to demonstrate the effectiveness of the WENO scheme in which the results are in good agreement with the analytical counterparts.
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1. Introduction

In this paper, we extend a multi-class Lighthill–Whitham–Richards (LWR) traffic flow model [21,23,28] to deal with inhomogeneous road conditions. The variable road conditions are the number of lanes $a(x)$ and the free flow (maximum) velocities $\{v_{l,f}(x)\}_{l=1}^{m}$ of m types of vehicles. Let $\rho_l(x,t)$ be the density per lane of the lth type, and let

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$$\rho(x,t) = \sum_{l=1}^{m} \rho_l(x,t)$$

be the total density per lane. The velocity of the lth type of vehicles is a function of ρ, which is denoted by $v_l(\rho)$. Furthermore, we assume that $\{v_l\}_{l=1}^m$ are related by

$$v_l = b_l(x)v(\rho), \quad v'(\rho) < 0, \quad b_l(x) = v_{l,f}(x)/v_f, \quad v_f \equiv \max_x \max_{1\leqslant l\leqslant m}(v_{l,f}(x)), \tag{1.1}$$

where the free flow velocity v_f is the maximum of the free flow velocities $v_{l,f}(x)$ of the lth type at location x. Accordingly, the velocity differences between m vehicle types are reflected by the functions $\{b_l(x)\}_{l=1}^m$ and $0 \leqslant b_l(x) \leqslant 1$.

The model equations are acquired from the mass conservation of m types of vehicles, which read

$$(a(x)\rho_l)_t + (a(x)\rho_l b_l(x)v(\rho))_x = 0, \quad 1 \leqslant l \leqslant m. \tag{1.2}$$

Eq. (1.2) is a natural extension of the so called multi-class LWR model that was proposed in [21] and studied in [23,28]. The present model reduces to that in [21,23,28] when $a(x)$ and $\{v_l\}_{l=1}^m$ are constants. We introduce the conservative solution variables $u_l = a(x)\rho_l$, the vector $u = (u_1,\ldots,u_m)^{\mathrm{T}}$, and the flux vector $f = (f_1,\ldots,f_m)^{\mathrm{T}}$ with $f_l = b_l u_l v(\Sigma u_l/a)$. Accordingly, the model equations can be written as

$$u_t + f(u,\theta(x))_x = 0, \tag{1.3}$$

where the vector function $\theta(x)$ represents all inhomogeneous factors on the road, namely,

$$\theta(x) = (a(x), b_1(x), \ldots, b_m(x)).$$

In this traffic flow problem, each density ρ_l and the total density ρ are bounded by a jam density ρ_{jam}, and thus

$$u/a \in \bar{D}, \bar{D} = \left\{ u/a | \rho_l \geqslant 0, \qquad l = 1,\ldots,m; \quad \sum_{l=1}^{m} \rho_l \leqslant \rho_{\text{jam}} \right\}. \tag{1.4}$$

Moreover, the function $v(\rho)$ of (1.1) satisfies

$$v(0) = v_f, \quad v(\rho_{\text{jam}}) = 0.$$

The study of this extended traffic flow system is significant both for practical application and theoretical interest. In real traffic, the drop or increase in traffic capacity that is reflected by $\theta(x)$ is frequent in many locations, such as on curves and slopes and near ramps and traffic accidents. In particular, by extension $b_l = b_l(x,t)$ can serve as a switch function in signal traffic or the like (see Section 4.2 for this extension). These changes are usually very sharp, thus, all the coefficients in θ are treated as being discontinuous at the change. In other words, the flux $f(u,\theta(x))$ is a discontinuous function of location x through the discontinuous function $\theta(x)$. When the extension $b_l = b_l(x,t)$ is considered, the flux $f(u,\theta(x,t))$ is a discontinuous function of location x and time t through the discontinuous function $\theta(x,t)$. Another complication comes from the fact that it is usually impossible (for $m > 2$) to solve the eigen-polynomial of system (1.3) explicitly, let alone the solutions to Riemann problems. One would thus be limited to use very crude approximate Riemann solvers such as the Lax–Friedrichs solvers for numerical schemes. Hence, first or even second-order numerical methods will be very dissipative. These together pose significant difficulties for both analytical and numerical studies. See for example [1,2,9,13,14,24–27] for related discussions.

In this paper, some important features of the model are discussed under a modified equivalent system of (1.3), in which all of the components of θ are solution variables. Analytically, the hyperbolicity of the system is proven, and the wave-breaking patterns of the Riemann problem are predicted. We note that these descriptions are mostly based on the relevant studies in [24,28]. The maximum absolute value of

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