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An efficient hybrid MLFMA–FFT solver for the volume integral equation in case of sparse 3D inhomogeneous dielectric scatterers

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Abstract

Electromagnetic scattering problems involving inhomogeneous objects can be numerically solved by applying a Method of Moments discretization to the volume integral equation. For electrically large problems, the iterative solution of the resulting linear system is expensive, both computationally and in memory use. In this paper, a hybrid MLFMA–FFT method is presented, which combines the fast Fourier transform (FFT) method and the High Frequency Multilevel Fast Multipole Algorithm (MLFMA) in order to reduce the cost of the matrix–vector multiplications needed in the iterative solver. The method represents the scatterers within a set of possibly disjoint identical cubic subdomains, which are meshed using a uniform cubic grid. This specific mesh allows for the application of FFTs to calculate the near interactions in the MLFMA and reduces the memory cost considerably, since the aggregation and disaggregation matrices of the MLFMA can be reused. Additional improvements to the general MLFMA framework, such as an extention of the FFT interpolation scheme of Sarvas et al. from the scalar to the vectorial case in combination with a more economical representation of the radiation patterns on the lowest level in vector spherical harmonics, are proposed and the choice of the subdomain size is discussed. The hybrid method performs better in terms of speed and memory use on large sparse configurations than both the FFT method and the HF MLFMA separately and it has lower memory requirements on general large problems. This is illustrated on a number of representative numerical test cases.

Keywords: Volume integral equation; Method of moments; FFT method; HF MLFMA; Vector spherical harmonics; Sparse geometries; Hybrid technique; Electromagnetic scattering

1. Introduction

In several research domains, it is necessary to simulate the full-vectorial, three-dimensional scattering of electromagnetic waves from inhomogeneous dielectric objects. A few examples are the calculation of radar cross

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sections of composite structures, the homogenization of meta materials and the reconstruction of objects with inhomogeneous permittivity from scattering data. A volume integral equation (VIE) formulation provides a solid framework for such scattering simulations. Typically, a method of moments (MoM) discretization is used to obtain an N-dimensional linear system which is then solved numerically. However, if the electrical size of the scatterers increases, inverting directly this $N \times N$-system becomes very slow due to the $\mathcal{O}(N^3)$ computational complexity. Moreover, the memory needed to store the system's $N \times N$-matrix can easily exceed the available computer memory. Solving the system iteratively using, for example, a conjugate gradient (CG) or stabilized bi-conjugate gradient (BICGSTAB) solver reduces the computational complexity to $\mathcal{O}(N^{\mathrm{I}}N^2)$, with N^{I} the number of iterations, but the system's matrix still has to be stored in this approach.

Two classes of methods that reduce the storage requirements and speed up the matrix–vector multiplications needed in every step of the iterative solver are the Fast Fourier transform (FFT) based techniques and the Multilevel Fast Multipole algorithms (MLFMA's). The first class [1–5] uses the FFT to exploit the convolutional structure of the integral operator in the VIE and has a computational complexity (for one matrix–vector multiplication) of $\mathcal{O}(N \log N)$ and a memory use of $\mathcal{O}(N)$. The methods in the second class, such as the High Frequency (HF) MLFMA [6–9], the Low Frequency MLFMA [10] and the Inhomogeneous or Stable Plane wave method [11,12], are based on efficient decompositions of the Green function. The major advantages of the FFT methods are their speed and easy implementation, thanks to the fast, reliable and widespread codes for calculating FFTs [13]. The MLFMA's on the other hand allow a more flexible meshing of the scattering geometry, since they can be applied to arbitrary meshes. For moderate to large volumetric problems with densely distributed mesh elements, the FFT methods are usually faster, thanks to their small prefactor, despite the lower computational complexity of MLFMA's ($\mathcal{O}(N)$) on such dense geometries.

In this paper, a hybrid MLFMA–FFT method is proposed, which is particularly suited for large scattering configurations that show some sparsity. The method is a modification of the HF MLFMA that treats the interactions between nearby mesh elements using FFTs and the interactions between well separated elements as in a regular HF MLFMA. It can also be regarded as a hybridization of the subdomain FFT method, which is proposed here as an FFT method for a collection of cubic subdomains. The subdomain meshing avoids the extension of the FFT grid over empty space between scatterers, as is necessary in the classical FFT method. It will be shown that the MLFMA–FFT method outperforms both the regular HF MLFMA and the FFT method on large sparse geometries and that it can have lower memory requirements even on large dense geometries.

The outline of this paper is as follows. In Section 2 the 3D scattering problem is formulated using a VIE and discretized with a MoM scheme. Section 3 proposes a subdomain FFT method to speed up the matrix–vector products needed for the iterative solution of the linear system. Section 4 starts by shortly revisiting the HF MLFMA. For a more thorough treatment, the reader is referred to [9]. Next, some improvements to the general MLFMA framework are presented. Specifically, the exploitation of symmetries in the subdomain mesh allows for a reduction of the memory cost of the MLFMA and the application of an FFT interpolation scheme for the vectorial MLFMA and the use of vector spherical harmonics to represent the radiation patterns on the lowest level result in accurate and efficient aggregation and disaggregation stages. Section 5 presents the hybrid MLFMA–FFT and discusses its relation to the FFT method and the HF MLFMA. Finally, several numerical examples are given in Section 6 to validate the method and to demonstrate its accuracy and superior performance.

Throughout this paper, we will work in the frequency regime and the time dependency $e^{j\omega t}$, with ω the angular frequency, will be implicitly assumed.

2. Problem formulation

2.1. 3D volume integral equation

Consider a number of 3D inhomogeneous, possibly lossy, dielectric objects with arbitrary shape that are situated in a homogeneous background medium with complex permittivity ϵ_{b} (Fig. 1). All materials are non-magnetic and have permeability μ_0. The scatterers are characterized by the complex permittivity $\epsilon(\boldsymbol{r}) = \epsilon'(\boldsymbol{r}) - j\epsilon''(\boldsymbol{r})$,