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Fast intersections on nested tetrahedrons (FINT): An algorithm for adaptive finite element based distributed parameter estimation

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Abstract

A variety of biomedical imaging techniques such as optical and fluorescence tomography, electrical impedance tomography, and ultrasound imaging can be cast as inverse problems, wherein image reconstruction involves the estimation of spatially distributed parameter(s) of the PDE system describing the physics of the imaging process. Finite element discretization of imaged domain with tetrahedral elements is a popular way of solving the forward and inverse imaging problems on complicated geometries. A dual-adaptive mesh-based approach wherein, one mesh is used for solving the forward imaging problem and the other mesh used for iteratively estimating the unknown distributed parameter, can result in high resolution image reconstruction at minimum computation effort, if both the meshes are allowed to adapt independently. Till date, no efficient method has been reported to identify and resolve intersection between tetrahedrons in independently refined or coarsened dual meshes. Herein, we report a fast and robust algorithm to identify and resolve intersection of tetrahedrons within nested dual meshes generated by 8-similar subtetrahedron subdivision scheme. The algorithm exploits finite element weight functions and gives rise to a set of weight functions on each vertex of disjoint tetrahedron pieces that completely cover up the intersection region of two tetrahedrons. The procedure enables fully adaptive tetrahedral finite elements by supporting independent refinement and coarsening of each individual mesh while preserving fast identification and resolution of intersection. The computational efficiency of the algorithm is demonstrated by diffuse photon density wave solutions obtained from a single- and a dual-mesh, and by reconstructing a fluorescent inclusion in simulated phantom from boundary frequency domain fluorescence measurements.

Keywords: Intersection; Tetrahedral mesh; Dual-mesh; Refinement; Derefinement; Tomography; Adaptive finite element; Fluorescence; Photon diffusion

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1. Introduction

Over the past few decades, electrical impedance, microwave computed, diffuse optical, and fluorescence enhanced optical tomographies have been sought as medical imaging techniques that use model-based, iterative reconstruction algorithms to reconstruct distinct tissue properties for identification of interior, diseased tissue from boundary value measurements. In these imaging modalities, the finite element method (FEM) is used to represent arbitrary tissue volumes for solution of the forward problem, i.e., prediction of the boundary measurements from a model and a given (or guessed) tissue property map, and for solution of the inverse problem, i.e. recovery of the interior tissue property map from a model and set of boundary measurements. The inverse imaging solution depends upon optimization procedures that seek to minimize the error between forward FEM predictions and actual measurements by iteratively adjusting the spatially distributed parameter of interest. In tomography applications, unknown parameter is typically discretized by nodal basis functions. However, other discretization schemes such as piecewise discontinuous parameter maps are also possible. The accuracy of the forward imaging problem solution along with the computation expense, increases with the refinement of the finite element mesh. For the inverse problem, increasing mesh refinement results in better potential image resolution, while at the same time increasing the number of unknown variables which can create numerical instability. Goal-based adaptive local refinement of finite element meshes can produce highly resolved images at a low computational cost.

The basis for mesh refinement for optimal forward and inverse problems differs and hence using a single mesh for solving the forward problem accurately as well as limiting the number of unknowns in the inverse problem is not possible. Therefore, the use of separate meshes, i.e., a refined mesh for accurate forward solutions and a coarse inverse mesh for parameter recovery enables a complete decoupling of the two problems. Based upon this recognition, fixed dual-mesh-based approaches have been proven to be successful [1–7]. However, when there is no *a priori* information available on the spatial distribution of tissue properties, the discretization of forward and inverse meshes becomes arbitrary rather than optimal.

Adaptive mesh refinements based upon *a posteriori* error estimates have been tailored to specific models [8–16] but have not been widely applied to distributed parameter estimation problems of the type encountered in tomography. The few reports that employ adaptive mesh refinement in two- and three-dimensional electrical impedance tomography [17–19] are based upon a single mesh used for both forward and inverse problems. Although the adaptive dual-mesh techniques employing refinement both in the forward and the inverse meshes were attempted for optical tomography [20], the work was limited to the two-dimensional problems. Recently, Joshi et al. [21,22] reported the use of a fully adaptive, three-dimensional fluorescence enhanced optical tomography technique that used a hexahedron-based dual-mesh scheme to improve image resolution at feasible computational cost. The approach employed a forward mesh that was refined/coarsened based upon the spatial gradient of excitation and emission fluences and of tissue properties while the inverse mesh was independently refined/coarsened based upon the spatial gradient of tissue fluorophore concentration. The use of hexahedral elements, however, limits application to simple rectilinear geometries. Curved geometries require higher order or isoparametric brick shaped elements resulting in complex algorithms. As tetrahedral meshes can be relatively easily generated for complicated geometries encountered in medical imaging, optimal, three-dimensional adaptive tomography employing tetrahedron-based dual-adaptive mesh scheme will positively impact multiple biomedical imaging modalities. However, independent refinement of tetrahedral elements in the two meshes requires solution to a computationally intensive intersection problem between two tetrahedrons on the different meshes, which prohibits its deployment in iterative parameter estimation algorithms. In principle, the intersection between two tetrahedrons can be found using a series of triangle-line piercing tests conducted in three-dimensional real object space. However, the object space intersection scheme is not robust due to the finite precision floating point arithmetic and thus sophisticated implementation is required to circumvent the precision errors. Furthermore, geometrical searching is required to pick candidate tetrahedrons for intersection in the two related forward and inverse meshes. However, the simple brute force algorithm employing intersection check for all possible pairs of tetrahedrons in the two meshes will cost $O(N^2)$ operations where N is the number of tetrahedrons. For static tetrahedral dual-mesh tomography, the intersection is computed once and stored at the preprocessing step. For iterative reconstruction phase, the intersection information thus stored is repetitively used. However, the adaptive dual-mesh tomography