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Construction of low dissipative high-order well-balanced filter schemes for non-equilibrium flows

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ABSTRACT

The goal of this paper is to generalize the well-balanced approach for non-equilibrium flow studied by Wang et al. (2009) [29] to a class of low dissipative high-order shock-capturing filter schemes and to explore more advantages of well-balanced schemes in reacting flows. More general 1D and 2D reacting flow models and new examples of shock turbulence interactions are provided to demonstrate the advantage of well-balanced schemes. The class of filter schemes developed by Yee et al. (1999) [33], Sjögreen and Yee (2004) [27] and Yee and Sjögreen (2007) [38] consist of two steps, a full time step of spatially high-order non-dissipative base scheme and an adaptive non-linear filter containing shock-capturing dissipation. A good property of the filter scheme is that the base scheme and the filter are stand-alone modules in designing. Therefore, the idea of designing a well-balanced filter scheme is straightforward, i.e. choosing a well-balanced base scheme with a well-balanced filter (both with high-order accuracy). A typical class of these schemes shown in this paper is the high-order central difference schemes/predictor–corrector (PC) schemes with a high-order well-balanced WENO filter. The new filter scheme with the well-balanced property will gather the features of both filter methods and well-balanced properties: it can preserve certain steady-state solutions exactly; it is able to capture small perturbations, e.g. turbulence fluctuations; and it adaptively controls numerical dissipation. Thus it shows high accuracy, efficiency and stability in shock/turbulence interactions. Numerical examples containing 1D and 2D smooth problems, 1D stationary contact discontinuity problem and 1D turbulence/shock interactions are included to verify the improved accuracy, in addition to the well-balanced behavior.

1. Introduction

Recent progress in the development of a class of low dissipative high-order filter schemes for multiscale Navier–Stokes and magnetohydrodynamics (MHD) systems [33,39,27,35,26,36–38] shows good performance in multiscale shock/turbulence simulations.

The highly parallelizable high-order filter methods consist of two steps, a full time step of spatially high-order non-dissipative (or very low dissipative) base scheme and an adaptive multistep filter. The non-linear filter consists of the product of a wavelet-based flow sensor and the dissipative portion of a high-order shock-capturing scheme. The

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built-in flow sensors in the post processing filter control the amounts and types of numerical dissipation. The filter switches on the dissipations only where needed, and leaves the rest of the flow region free from numerical dissipation. Only the filter step might involve the use of flux limiters and approximate Riemann solvers as stabilizing mechanisms to remove Gibbs phenomena related spurious oscillations resulting from the base scheme step. The more scales that are resolved by the base scheme, the less the filter is utilized, thereby gaining accuracy and computational time as the grid is refined. The adaptive numerical dissipation control idea is very general and can be used in conjunction with spectral, spectral element, finite element, discontinuous Galerkin, finite volume, and finite difference spatial base schemes. The type of shock-capturing schemes used as non-linear dissipation can be the dissipative portion of any high-resolution TVD, MUSCL, ENO, or WENO shock-capturing method [33,13,23]. By design, flow sensors, spatial base schemes and linear and non-linear dissipation models are stand-alone modules. Therefore, a whole class of low dissipative high-order schemes can be derived at ease.

In a recent paper by Wang et al. [29], well-balanced finite difference WENO schemes and second-order TVD schemes were studied for chemical non-equilibrium flows, extending the well-balanced finite difference WENO schemes for shallow water equations in [30,31]. A well-balanced scheme [7,15,5], which can preserve certain non-trivial steady-state solutions exactly, may help minimize some of the spurious oscillations around steady states. It was also shown in [29] that the well-balanced schemes capture small perturbations of the steady-state solutions with high accuracy. While general schemes can only resolve perturbations at the level of truncation error with the specific grid, well-balanced schemes can resolve much smaller perturbations, usually of size 1% or lower of the main steady-state flow.

In this paper the same approach will be applied to construct a high-order well-balanced filter scheme for one temperature non-equilibrium flow with reaction terms. The multi-dimensional hyperbolic system of conservation laws with source terms (also called a balance law)

$$U_t + \nabla \cdot F(U) = S(U) \tag{1}$$

is considered, where U is the solution vector, $F(U)$ is the convective flux and $S(U)$ is the source term. For this type of flow the space variable $\mathbf{x}$ does not appear explicitly in the source term. Thus, the construction of well-balanced schemes can easily go from one-dimension to multi-dimensions. In this paper, comparing with our earlier work [29], more general 1D and 2D reacting flow models and new examples of shock turbulence interactions are provided to demonstrate the advantage of well-balanced schemes.

The designing of well-balanced filter schemes is to choose a well-balanced base scheme and a well-balanced filter part. Then, the filter scheme is almost well-balanced everywhere except at the interfaces of the filtered region and the non-filtered region (see Section 4). Note that in this paper, a 'well-balanced filter scheme' refers to such almost well-balancedness. For the filter scheme without the flow sensor, the resulting filter scheme is well-balanced.

The choice of the sensor will not destroy the well-balanced properties. It has been shown in the previous work [29] that linear schemes, the second-order Predictor–Corrector (PC) scheme [34,16] with TVD filters (such as the Harten–Yee TVD filter [32,33]), and WENO-Roe schemes are well-balanced for certain steady-state solutions with zero velocity. A well-balanced WENO-LF scheme has also been constructed for this type of steady-state solutions. High-order PC schemes are linear schemes and thus well-balanced. Therefore, the new filter schemes presented in this paper, CENTVDfi/CENWENOfi or PCTVDfi/PCWENOfi which utilize central (CEN)/PC schemes as base schemes and the Harten–Yee TVD filter (TVDfi) or the well-balanced WENO schemes as filter (WENOfi) will be well-balanced. We remark that this paper is confined mainly to the spatial discretizations. Appropriate time discretizations should be an integral part of the algorithm.

In this paper, only the zero-velocity steady state of the reacting flow equations will be considered in the numerical tests. A steady state with zero velocity implies that the flow has constant pressure and is in chemical equilibrium. It will be shown that, similarly to well-balanced WENO schemes, well-balanced filter schemes give machine round-off errors regardless of the mesh sizes for the steady-state solutions of the reactive flow equations. Consequently, they can resolve small perturbations of such steady-state solutions well, even with very coarse meshes.

Since the regular high-order low dissipative filter schemes are designed for shock/turbulence interactions and the well-balanced schemes can capture small perturbations of the steady-state solutions with high accuracy, the new well-balanced filter schemes take the advantages of both, thereby making them well suited for computations of turbulent fluctuations on a mainly steady flow field.

The outline of the paper is as follows: in Section 2, the governing equations and the physical model are described. High-order filter schemes are reviewed in Section 3. A brief description of well-balanced schemes and the construction of high-order well-balanced filter scheme are given in Section 4. Numerical examples will be shown in Section 5. Finally, Section 6 gives conclusions and plans for future work. A brief description of high-order PC schemes and the considered time discretization are presented in the Appendix.

2. Governing equations and the physical model

Considering a flow in chemical non-equilibrium and thermal equilibrium, the thermodynamic properties account for excitation of the electronic states for the atoms and molecules, and rovibrational states based on the rigid-rotor harmonic-oscillator approximation for molecules [17,20].