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An implicit method for coupled flow–body dynamics

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Abstract

We propose an efficient method for computing coupled flow–body dynamics. The time-stepping is implicit, and uses an iterative method (preconditioned GMRES) to solve the flow–body equations. The preconditioner solves a decoupled version of the equations which involves only the inversion of banded matrices, and requires a small number of iterations per time step. We use the method to probe the instability to horizontal motions of an elliptical body with simple vertical motions: flapping and rising. In both cases a linear instability to horizontal motion sets in above a critical Reynolds number, leading to a stable oscillatory state. The pressure forces play a destabilizing role against the stabilizing viscous forces, with oscillatory time scales set by either external flapping or the intrinsic flow–body coupling. The latter lowers the instability threshold in Reynolds number.

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1. Introduction

Many problems of recent interest in biolocomotion involve the dynamics of fully-coupled flow–body systems [1–4]. When the motion of a fluid–solid boundary is not prescribed, but is instead determined in terms of the fluid forces on it, traditional methods for computational fluid dynamics face additional stability constraints.

Here we consider unsteady two-dimensional incompressible fluid flows, described by the Navier–Stokes equations. Among the many formulations used in numerical schemes, we focus on the finite-difference “vorticity–stream-function” formulation, one of the most widely-used. Over the past 80 years, most discussions of this formulation in the literature deal with the appropriate method for imposing the velocity boundary conditions (no-slip and no-penetration) on the vorticity–stream-function system [5]. The discretized boundary conditions exert a determining influence on both the accuracy and stability of the scheme, and often the former is decreased to enhance the latter [6]. For explicit schemes, stability requires that the time step be less than

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a constant times the square of the smallest mesh spacing (the CFL condition), times the Reynolds number. Such schemes have recently been developed to study high-Reynolds-number flows [7] ($Re = 10^3 - 10^6$), and used in studies of flapping flight [8]. Typically the time step is $10^{-6} - 10^{-4}$ in these studies, where 1 is the time scale of external forcing (such as the flapping period of a wing). Thus a large computational expense is required to compute only a few flapping periods. When the motion of the body is not prescribed as in the above studies, but coupled to fluid forces, the CFL bound on the time step becomes even more strict, now involving the cube, not the square, of the smallest mesh spacing. Anderson et al. studied the falling of a plate coupled to a viscous fluid using an explicit scheme [9]. The dynamical properties of the system were predicted based on a small number of tumbling periods, but longer runs would have made the results more conclusive.

In such cases the time step required for stability is many orders of magnitude smaller than that required for high accuracy, and implicit schemes become more attractive. Such schemes involve the solution of a sparse matrix equation representing the coupled system with derivatives discretized using finite differences. As the matrix is sparse, iterative methods are generally more efficient than a direct solution, particularly when an effective preconditioner can be found.

Here we propose an implicit, iterative method for computing flow–body interactions. The amount of work required for each iteration scales as the number of grid points, which is of the same order as the work required for one time step of an explicit scheme [7]. Hence the implicit scheme is more efficient than the explicit scheme when the number of time steps decreases by a larger factor than the number of iterations. In our method, the number of iterations per time step ranges from 20 to 100, while the size of stable time steps can increase by a factor of 10^4 or more, resulting in a decrease in computational time of a factor of more than 100. We give results of the method for studies of flow–body interactions at moderate Reynolds number ($10–10^2$), which are useful for understanding the behavior of bodies at the transition from low- to high-Reynolds number fluid dynamics [10], including the locomotion of small insects [11] and copepods [12]. However, the method is not inherently limited to this range of Reynolds number.

In Section 2, we give the vorticity–stream-function equations and boundary conditions in an infinite plane, written in terms of a boundary-fitted elliptic mesh. In Section 3, we give the implicit iterative scheme, which is designed to be closely approximated by a preconditioner matrix which is easy to solve. We use an operator splitting to write the unsteady and viscous terms as pentadiagonal matrices. Essential to the rapid convergence of the iteration is the preconditioner, which approximates the full system of equations by a decoupled block-matrix system. Hence the equations coupling boundary vorticity to the bulk flow, and to the motion of the body, are omitted in the preconditioner. Nonetheless, we obtain convergence in a small number of iterations.

In Section 4, we give results for the dynamics of a wing with motion coupled to the ambient flow, over times of up to 100 flapping periods, and compare these dynamics with those of an untethered bluff body in a steady flow. Both cases are near the Reynolds number at which instability to transverse oscillations sets in. We present a detailed picture of the instability, including the destabilizing role of pressure forces versus the stabilizing role of viscous forces, and the presence of phase-locking. We also find a transition from regular oscillations to irregular motions as the Reynolds number increases. In the appendices we present convergence studies and validations of the current method using benchmark problems.

2. Coupled flow–body equations

We solve the 2-D Navier–Stokes equations for the motion of an incompressible, viscous fluid in the vorticity–stream-function formulation. These are written in terms of the vorticity ω, the flow velocity $\mathbf{u} = u_1\hat{\mathbf{e}}_x + u_2\hat{\mathbf{e}}_y$ and the stream function ψ

$$\frac{\partial\omega}{\partial t} + \mathbf{u}\cdot\nabla\omega = \frac{1}{Re}\Delta\omega,$$

$$\Delta\psi = \omega,$$

$$\mathbf{u} = \nabla^{\perp}\psi = (-\partial_y, \partial_x)\psi.$$

We have given the equations in standard nondimensional form using the Reynolds number $Re = LU/\nu$ with the characteristic velocity U and distance L defined below for two specific problems. We formulate our method